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Name of the subject : DESIGN OF REINFORCED CONCRETE & BRICK MASONRY STRUCTURES.		Design of RC Elements, Mr. N . krishnaraju
Date of deliverance :		

Unit III – SELECTED TOPICS

Design of staircases (ordinary and doglegged) – Design of flat slabs – Design of Reinforced concrete walls – Principles of design of mat foundation-Introduction to prestressed concrete-Principles –types and methods of prestressing- BIS Codal Provisions

Stairs are a system of steps that allows the passage of people and objects from one level to another. They can be made from timber, concrete and sometimes steel or stone aluminium and with modern technology and materials even glass.

Types of Staircases

Figures

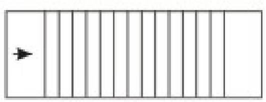


Fig. 9.20.1(a): Single flight staircase

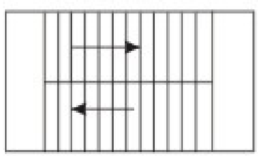


Fig. 9.20.1(b): Two flight staircase

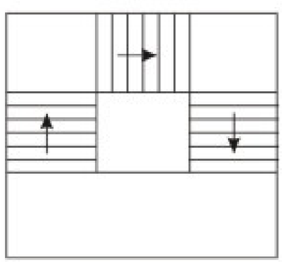


Fig. 9.20.1 (c): Open-well staircase

Fig. 9.20.1: Types of staircases

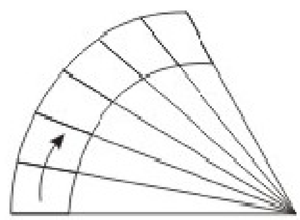


Fig. 9.20.1(e): Helicoidal staircase

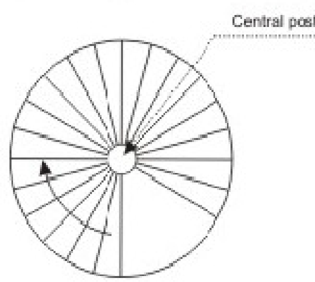


Fig. 9.20.1(d): Spiral staircase

Fig. 9.20.1: Types of staircases

Figures 9.20.1a to e present some of the common types of staircases based on geometrical configurations:

(a) Single flight staircase (Fig. 9.20.1a)

(b) Two flight staircase (Fig. 9.20.1b)

(c) Open-well staircase (Fig. 9.20.1c)

(d) Spiral staircase (Fig. 9.20.1d)

(e) Helicoidal staircase (Fig. 9.20.1e)

Architectural considerations involving aesthetics, structural feasibility and functional requirements are the major aspects to select a particular type of the staircase. Other influencing parameters of the selection are lighting, ventilation, comfort, accessibility, space etc.

Typical Flight

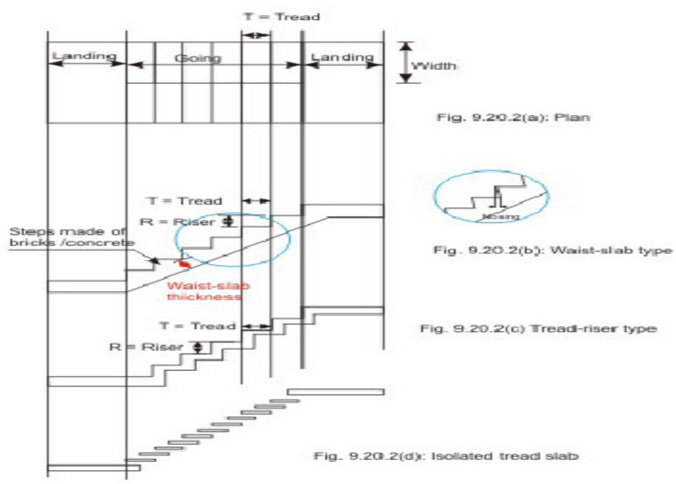


Fig. 9.20.2: A typical flight

Figures 9.20.2a to d present plans and sections of a typical flight of different possibilities.

The different terminologies used in the staircase are given below:

(a) Tread: The horizontal top portion of a step where foot rests (Fig.9.20.2b) is known as tread. The dimension ranges from 270 mm for residential buildings and factories to 300 mm for public buildings where large number of persons use the staircase.
(b) Nosing: In some cases the tread is projected outward to increase the space. This projection is designated as nosing (Fig.9.20.2b).
(c) Riser: The vertical distance between two successive steps is termed as riser (Fig.9.20.2b). The dimension of the riser ranges from 150 mm for public buildings to 190 mm for residential buildings and factories.
(d) Waist: The thickness of the waist-slab on which steps are made is known as waist (Fig.9.20.2b). The depth (thickness) of the waist is the minimum thickness perpendicular to the soffit of the staircase (cl. 33.3 of IS 456). The steps of the staircase resting on waist-slab can be made of bricks or concrete.
(e) Going: Going is the horizontal projection between the first and the last riser of an inclined flight (Fig.9.20.2a).

The flight shown in Fig.9.20.2a has two landings and one going. Figures 9.2b to d present the three ways of arranging the flight as mentioned below:

(i) waist-slab type (Fig.9.20.2b),
(ii) tread-riser type (Fig.9.20.2c), or free-standing staircase, and
(iii) isolated tread type (Fig.9.20.2d).

9.20.4 General Guidelines

The following are some of the general guidelines to be considered while planning a staircase:

- The respective dimensions of tread and riser for all the parallel steps should be the same in consecutive floor of a building.
- The minimum vertical headroom above any step should be 2 m.
- Generally, the number of risers in a flight should be restricted to twelve.
- The minimum width of stair (Fig.9.20.2a) should be 850 mm, though it is desirable to have the width between 1.1 to 1.6 m. In public building, cinema halls etc., large widths of the stair should be provided.

Structural Systems

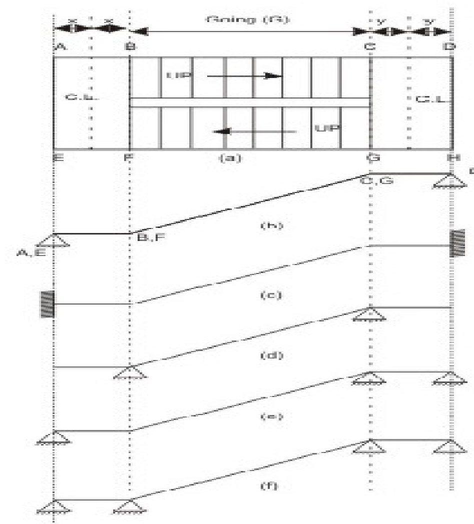


Fig. 9.20.3: Longitudinally spanning staircases

Different structural systems are possible for the staircase, shown in Fig. 9.20.3a, depending on the spanning direction. The slab component of the stair spans either in the direction of going i.e., longitudinally or in the direction of the steps, i.e., transversely. The systems are discussed below:

(A) Stair slab spanning longitudinally

Here, one or more supports are provided parallel to the riser for the slab bending longitudinally. Figures 9.20.3b to f show different support arrangements of a two flight stair of Fig.9.20.3a:

(i) Supported on edges AE and DH (Fig.9.20.3b)
(ii) Clamped along edges AE and DH (Fig.9.20.3c)
(iii) Supported on edges BF and CG (Fig.9.20.3d)
(iv) Supported on edges AE, CG (or BF) and DH (Fig.9.20.3e)
(v) Supported on edges AE, BF, CG and DH (Fig.9.20.3f)

Cantilevered landing and intermediate supports (Figs.9.20.3d, e and f) are helpful to induce negative moments near the supports which reduce the positive moment and thereby the depth of slab becomes economic.

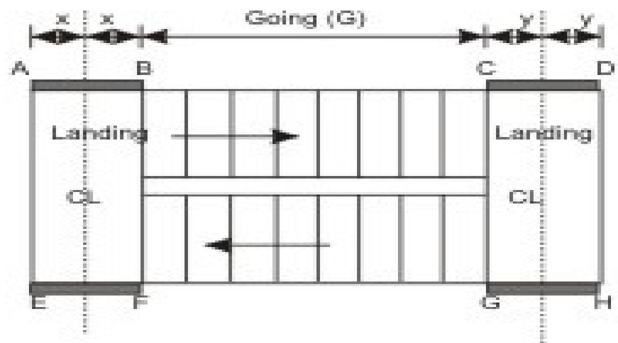


Fig. 9.20.4(a): Beams at two ends of landings

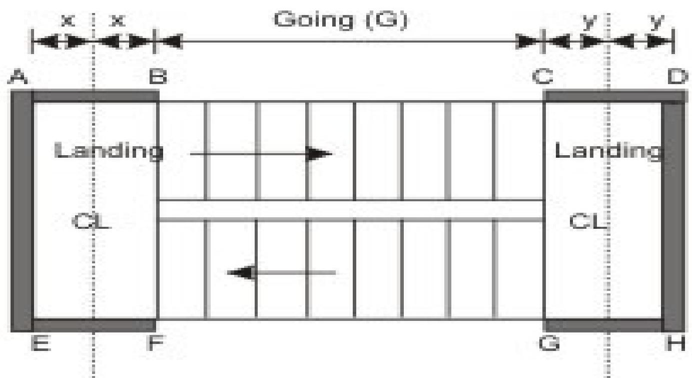


Fig. 9.20.4(b): Beams at three ends of landings

Fig. 9.20.4: Staircases (spanning longitudinally) and landings (spanning transversely)

In the case of two flight stair, sometimes the flight is supported between the landings which span transversely (Figs.9.20.4a and b). It is worth mentioning that some of the above mentioned structural systems are statically determinate while others are statically indeterminate where deformation conditions have to take into account for the analysis. Longitudinal spanning of stair slab is also possible with other configurations including single flight, open-well helicoidal and free-standing staircases.

(B) Stair slab spanning transversely

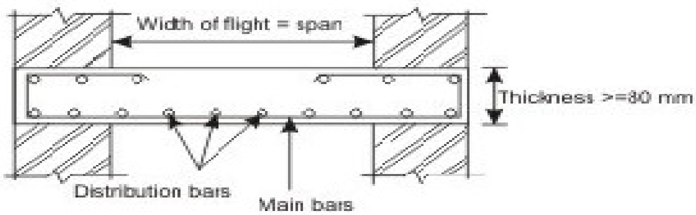


Fig. 9.20.5(a): Slabs supported between two stringer beams or walls

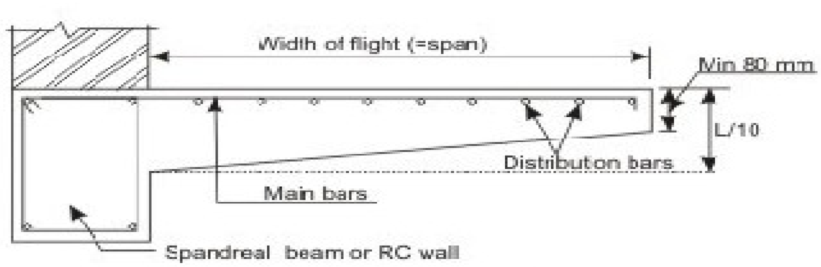


Fig. 9.20.5(b): Cantilever slab from a spandreal beam or wall

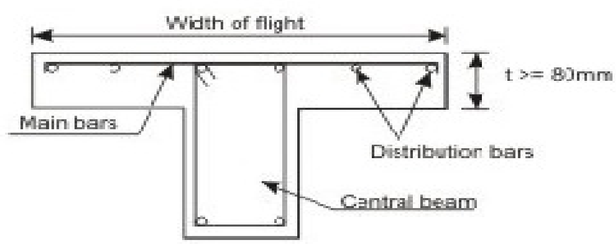


Fig. 9.20.5(c): Doubly cantilever slab from a central beam

Fig. 9.20.5: Transversely spanning staircases

Here, either the waist slabs or the slab components of isolated tread-slab and trade-riser units are supported on their sides or are cantilevers along the width direction from a central beam. The slabs thus bend in a transverse vertical plane. The following are the different arrangements:

(i) Slab supported between two stringer beams or walls (Fig.9.20.5a)

(ii) Cantilever slabs from a spandrel beam or wall (Fig.9.20.5b)

(iii) Doubly cantilever slabs from a central beam (Fig.9.20.5c)

Effective Span of Stairs

The stipulations of clause 33 of IS 456 are given below as a ready reference regarding the determination of effective span of stair. Three different cases are given to determine the effective span of stairs without stringer beams.

(i) The horizontal centre-to-centre distance of beams should be considered as the effective span when the slab is supported at top and bottom risers by beams spanning parallel with the risers.
(ii) The horizontal distance equal to the going of the stairs plus at each end either half the width of the landing or one meter, whichever is smaller when the stair slab is spanning on to the edge of a landing slab which spans parallel with the risers. See Table 9.1 for the effective span for this type of staircases shown in Fig.9.20.3a.

Table 9.1 Effective span of stairs shown in Fig.9.20.3a

Sl. No.	x	y	Effective span in meters
1	< 1 m	< 1 m	$G + x + y$
2	< 1 m	≥ 1 m	$G + x + 1$
3	≥ 1 m	< 1 m	$G + y + 1$
4	≥ 1 m	≥ 1 m	$G + 1 + 1$

Note: G = Going, as shown in Fig. 9.20.3a

9.20.7 Distribution of Loadings on Stairs

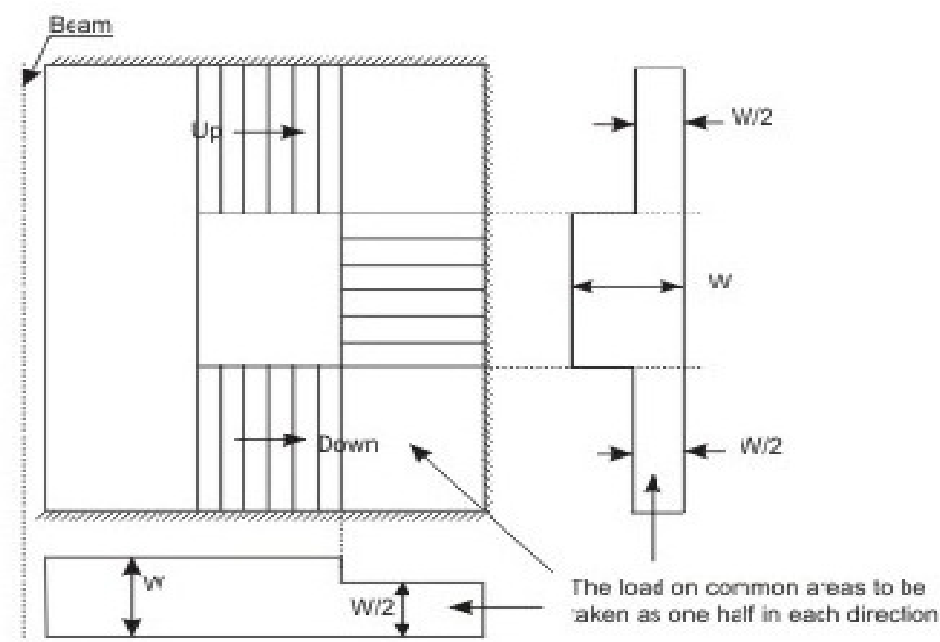


Fig. 9.20.6: Loadings on open-well staircases

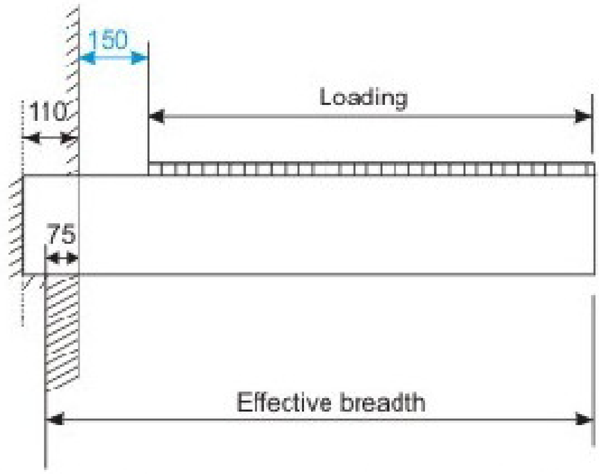


Fig. 9.20.7: Loading on staircases built into walls

Figure 9.20.6 shows one open-well stair where spans partly cross at right angle. The load in such stairs on areas common to any two such spans should be taken as fifty per cent in each direction as shown in Fig.9.20.7. Moreover, one 150 mm strip may be deducted from the loaded area and the effective breadth of the section is increased by 75 mm for the design where flights or landings are embedded into walls for a length of at least 110 mm and are designed to span in the direction of the flight (Fig.9.20.7).

9.20.8 Structural Analysis

Most of the structural systems of stair spanning longitudinally or transversely are standard problems of structural analysis, either statically determinate or indeterminate. Accordingly, they can be analyzed by methods of analysis suitable for a particular system. However, the rigorous analysis is difficult and involved for a trade-riser type or free standing staircase where the slab is repeatedly folded. This type of staircase has drawn special attraction due to its aesthetic appeal and, therefore, simplified analysis for this type of staircase spanning longitudinally is explained below. It is worth mentioning that certain idealizations are made in the actual structures for the applicability of the simplified analysis. The designs based on the simplified analysis have been found to satisfy the practical needs.

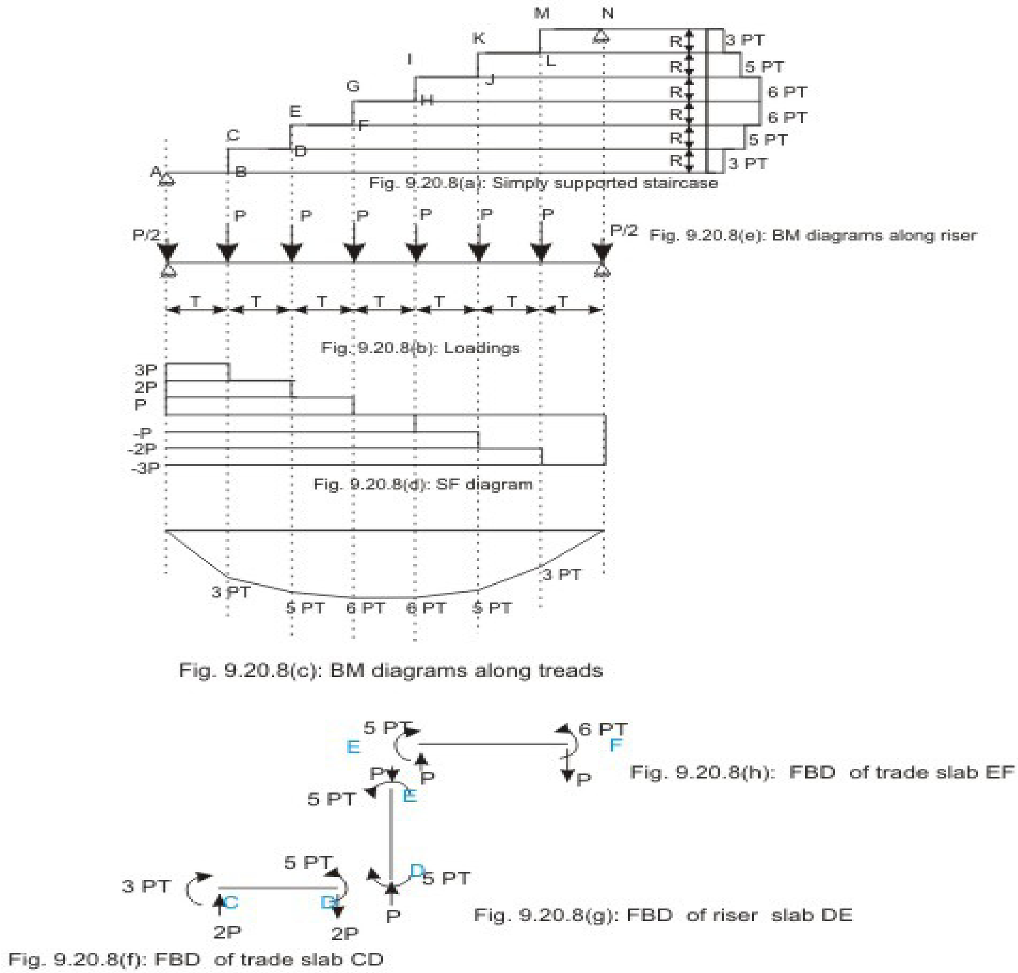


Fig. 9.20.8: Structural analysis of simply supported trade-riser staircase

Figure 9.20.8a shows the simply supported trade-riser staircase. The uniformly distributed loads are assumed to act at the riser levels (Fig.9.20.8b). The bending moment and shear force diagrams along the treads and the bending moment diagram along the risers are shown in Figs.9.20.8c, d and e, respectively. The free body diagrams of CD, DE and EF are shown in Figs.9.20.8f, g and h, respectively. It is seen that the trade slabs are subjected to varying bending moments and constant shear force (Fig.9.20.8f). On the other hand the riser slabs are subjected to a constant bending moment and axial force (either compressive or tensile). The assumption is that the riser and trade slabs are rigidly connected. It has been observed that both trade and riser slabs may be designed for bending moment alone as the shear stresses in trade slabs and axial forces in riser slabs are comparatively low. The slab thickness of the trade and risers should be kept the same and equal to span/25 for simply supported and span/30 for continuous stairs.

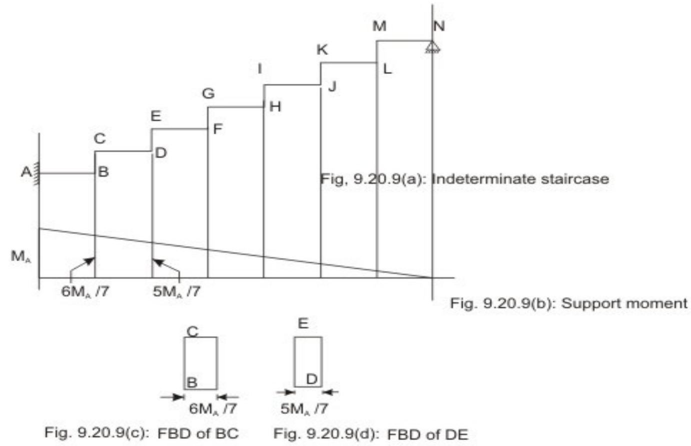


Fig. 9.20.9: Structural analysis of an indeterminate trade-riser staircase

Figure 9.20.9a shows an indeterminate trade-riser staircase. Here, the analysis can be done by adding the effect of the support moment M_A (Fig.9.20.9b) with the results of earlier simply supported case. However, the value of M_A can be determined using the moment-area method. The free body diagrams of two vertical risers BC and DE are show in Figs.9.20.9c and d, respectively.

9.20.9 Illustrative Examples

Two typical examples of waist-slab and trade-riser types spanning longitudinally are taken up here to illustrate the design.

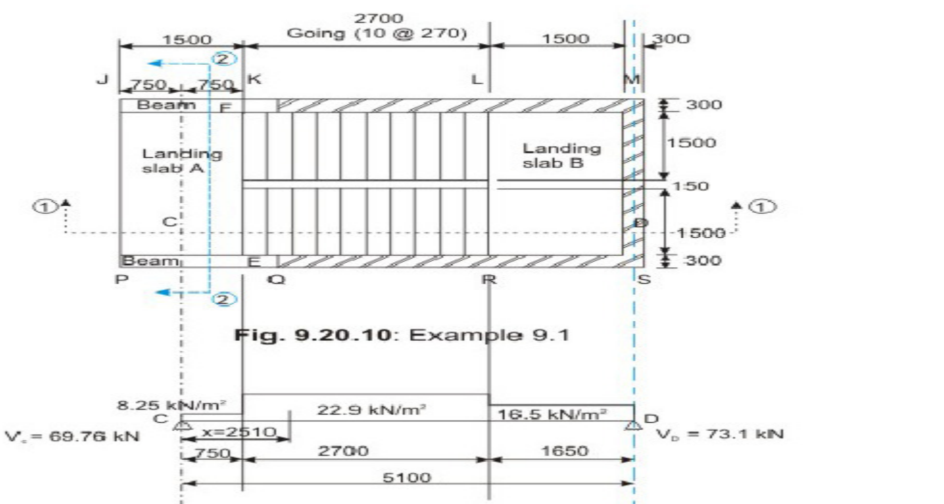


Fig. 9.20.10: Example 9.1

Fig. 9.20.11: Calculation of loads, sec 1-1 of Example 9.1, (Fig. 9.20.10)

Example 9.1:

Design the waist-slab type of the staircase of Fig.9.20.10. Landing slab A is supported on beams along JK and PQ, while the waist-slab and landing slab B are spanning longitudinally as shown in Fig.9.20.10. The finish loads and live loads are 1 kN/m^2 and 5 kN/m^2, respectively. Use riser R = 160 mm, trade T = 270 mm, concrete grade = M 20 and steel grade = Fe 415.

Solution:

With R = 160 mm and T = 270 mm, the inclined length of each step = $\{(160)^2 + (270)^2\}^{½}$ = 313.85 mm.

(A) Design of going and landing slab B

Step 1: Effective span and depth of slab

The effective span (cls. 33.1b and c) = 750 + 2700 + 1500 + 150 = 5100 mm.
The depth of waist slab = 5100/20 = 255 mm. Let us assume total depth of 250 mm and effective depth = 250 – 20 – 6 = 224 mm (assuming cover = 20 mm and diameter of main reinforcing bar = 12 mm). The depth of landing slab is assumed as 200 mm and effective depth = 200 – 20 – 6 = 174 mm.

Step 2: Calculation of loads (Fig.9.20.11, sec. 1-1)

(i) Loads on going (on projected plan area)

(a) Self-weight of waist-slab = 25(0.25)(313.85)/270 = 7.265 kN/m^2

(b) Self-weight of steps = 25(0.5)(0.16) = 2.0 kN/m^2

(c) Finishes (given) = 1.0 kN/m^2

(d) Live loads (given) = 5.0 kN/m^2

Total = 15.265 kN/m^2

Total factored loads = 1.5(15.265) = 22.9 kN/m^2
(ii) Loads on landing slab A (50% of estimated loads)

(a) Self-weight of landing slab = 25(0.2) = 5 kN/m^2

(b) Finishes (given) = 1 kN/m^2

(c) Live loads (given) = 5 kN/m^2

Total = 11 kN/m^2

Factored loads on landing slab A = 0.5(1.5)(11) = 8.25 kN/m^2

(iii) Factored loads on landing slab B = (1.5)(11) = 16.5 kN/m^2
The loads are drawn in Fig.9.20.11.

Step 3: Bending moment and shear force (Fig. 9.20.11)

Total loads for 1.5 m width of flight =1.5{8.25(0.75) + 22.9(2.7) + 16.5(1.65)}
= 142.86 kN

V_C = 1.5{8.25(0.75)(5.1 – 0.375) + 22.9(2.7)(5.1 – 0.75 – 1.35)
+ 16.5(1.65)(1.65)(0.5)}/5.1 = 69.76 kN
V_D = 142.86 – 69.76 = 73.1 kN
The distance x from the left where shear force is zero is obtained from:
x = {69.76 – 1.5(8.25)(0.75) + 1.5(22.9)(0.75)}/(1.5)(22.9) = 2.51 m
The maximum bending moment at x = 2.51 m is
= 69.76(2.51) – (1.5)(8.25)(0.75)(2.51 – 0.375)
- (1.5)(22.9)(2.51 – 0.75)(2.51 – 0.75)(0.5) = 102.08 kNm.
For the landing slab B, the bending moment at a distance of 1.65 m from D
= 73.1(1.65) – 1.5(16.5)(1.65)(1.65)(0.5) = 86.92 kNm

Step 4: Checking of depth of slab

From the maximum moment, we get $d = \{102080/2(2.76)\}^{½}$ = 135.98 mm < 224 mm for waist-slab and < 174 mm for landing slabs. Hence, both the depths of 250 mm and 200 mm for waist-slab and landing slab are more than adequate for bending.

For the waist-slab, $v\tau$ = 73100/1500(224) = 0.217 N/mm^2. For the waist-slab of depth 250 mm, k = 1.1 (cl. 40.2.1.1 of IS 456) and from Table 19 of IS 456, $c\tau$ = 1.1(0.28) = 0.308 N/mm^2. Table 20 of IS 456, max$c\tau$ = 2.8 N/mm^2. Since $v\tau < c\tau <$ max$c\tau$, the depth of waist-slab as 250 mm is safe for shear.

For the landing slab, $v\tau$ = 73100/1500(174) = 0.28 N/mm^2. For the landing slab of depth 200 mm, k = 1.2 (cl. 40.2.1.1 of IS 456) and from Table 19 of IS 456, $c\tau$ = 1.2(0.28) = 0.336 N/mm^2 and from Table 20 of IS 456, max$c\tau$ = 2.8 N/mm^2. Here also $v\tau < c\tau <$ max$c\tau$, so the depth of landing slab as 200 mm is safe for shear.

Step 5: Determination of areas of steel reinforcement

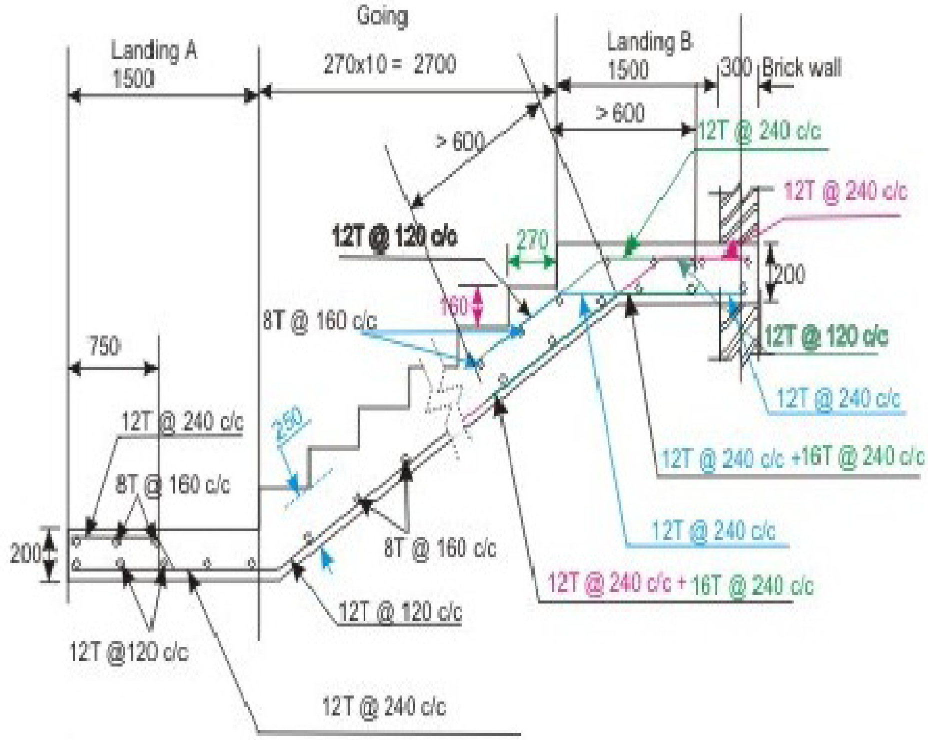


Fig. 9.20.12: Reinforcing bars of Example 9.1, sec 1-1 of Fig. 9.20.10

i) Waist-slab: M_u/bd^2 = 102080/(1.5)224(224) = 1.356 N/mm^2. Table 2 of SP-16 gives p = 0.411.

The area of steel = 0.411(1000)(224)/(100) = 920.64 mm^2.

Provide 12 mm diameter @ 120 mm c/c (= 942 mm^2/m).

(ii) Landing slab B: M_u/bd^2 at a distance of 1.65 m from V_D (Fig. 9.20.11)

= 86920/(1.5)(174)(174) = 1.91 N/mm^2. Table 2 of SP-16 gives: p = 0.606. The area of steel = 0.606(1000)(174)/100 = 1054 mm^2/m. Provide 16 mm diameter @ 240 mm c/c and 12 mm dia. @ 240 mm c/c (1309 mm^2) at the bottom of landing slab B of which 16 mm bars will be terminated at a distance of 500 mm from the end and will continue up to a distance of 1000 mm at the bottom of waist slab (Fig. 9.20.12).

Distribution steel: The same distribution steel is provided for both the slabs as calculated for the waist-slab. The amount is = 0.12(250) (1000)/100 = 300 mm^2/m. Provide 8 mm diameter @ 160 mm c/c (= 314 mm^2/m).

Step 6: Checking of development length and diameter of main bars

Development length of 12 mm diameter bars = 47(12) = 564 mm, say 600 mm and the same of 16 mm dia. Bars = 47(16) = 752 mm, say 800 mm.

(i) For waist-slab

M_1 for 12 mm diameter @ 120 mm c/c (= 942 mm^2) = 942(102.08)/920.64 = 104.44 kNm. With V (shear force) = 73.1 kN, the diameter of main bars ≤ {1.3(104440)/73.1}/47 39.5 mm. Hence, 12 mm diameter is o.k. ≤
(ii) For landing-slab B
M_1 for 16 mm diameter @ 120 mm c/c (= 1675 mm^2) = 1675(102.08)/1650.88 = 103.57 kNm. With V (shear force) = 73.1 kN, the diameter of main bars {1.3(103570)/73.1}/47 = 39.18 mm. Hence, 16 mm diameter is o.k. ≤
The reinforcing bars are shown in Fig.9.20.12 (sec. 1-1).

(B) Design of landing slab A

Step 1: Effective span and depth of slab

(i) The effective span is lesser of (i) (1500 + 1500 + 150 + 174), and (ii) (1500 + 1500 + 150 + 300) = 3324 mm. The depth of landing slab = 3324/20 = 166 mm, < 200 mm already assumed. So, the depth is 200 mm.

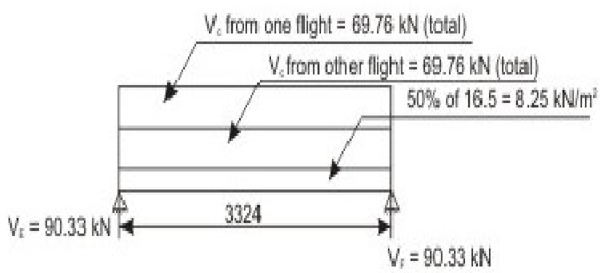


Fig. 9.20.13: Calculation of loads, sec 2-2 of Example 9.1, (Fig. 9.20.10)

Step 2: Calculation of loads (Fig.9.20.13)

The following are the loads:
(i) Factored load on landing slab A(see Step 2 of A @ 50%) = 8.25 kN/m^2
(ii) Factored reaction V_C (see Step 3 of A) = 69.76 kN as the total load of one flight
(iii) Factored reaction V_C from the other flight = 69.76 kN
Thus, the total load on landing slab A
= (8.25)(1.5)(3.324) + 69.76 + 69.76 = 180.65 kN

Due to symmetry of loadings, $V_E = V_F$ = 90.33 kN. The bending moment is maximum at the centre line of EF.

Step 3: Bending moment and shear force (width = 1500 mm)

Maximum bending moment = (180.65)(3.324)/8 = 75.06 kNm
Maximum shear force = 0.5(180.65) = 90.33 kN

Step 4: Checking of depth of slab

In Step 3 of A, it has been observed that 135.98 mm is the required depth for bending moment = 102.08 kNm. So, the depth of 200 mm is safe for this bending moment of 75.06 kNm. However, a check is needed for shear force.

$v\tau$ = 90330/1500(174) = 0.347 N/mm^2 > 0.336 N/mm^2

The above value of $c\tau$ = 0.336 N/mm^2 for landing slab of depth 200 mm has been obtained in Step 4 of A. However, here $c\tau$ is for the minimum tensile steel in the slab. The checking of depth for shear shall be done after determining the area of tensile steel as the value of $v\tau$ is marginally higher.

Step 5: Determination of areas of steel reinforcement

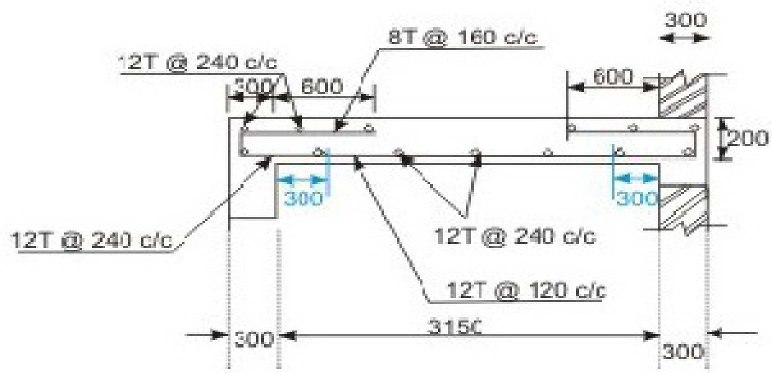


Fig. 9.20.14: Reinforcing bars of Example 9.1, sec 2-2 of Fig. 9.20.10

For M_u/bd^2 = 75060/(1.5)(174)(174) = 1.65 N/mm^2, Table 2 of SP-16 gives p = 0.512.

The area of steel = (0.512)(1000)(174)/100 = 890.88 mm^2/m. Provide 12 mm diameter @ 120 mm c/c (= 942 mm^2/m). With this area of steel p = 942(100)/1000(174) = 0.541.

Distribution steel = The same as in Step 5 of A i.e., 8 mm diameter @ 160 mm c/c.

Step 6: Checking of depth for shear

Table 19 and cl. 40.2.1.1 gives: $c\tau$ = (1.2)(0.493) = 0.5916 N/mm^2. $v\tau$ = 0.347 N/mm^2 (see Step 3 of B) is now less than $c\tau$ (= 0.5916 N/mm^2). Since, $v\tau < c\tau <$ max$c\tau$, the depth of 200 mm is safe for shear.

The reinforcing bars are shown in Fig. 9.20.14.

Example 9.2:

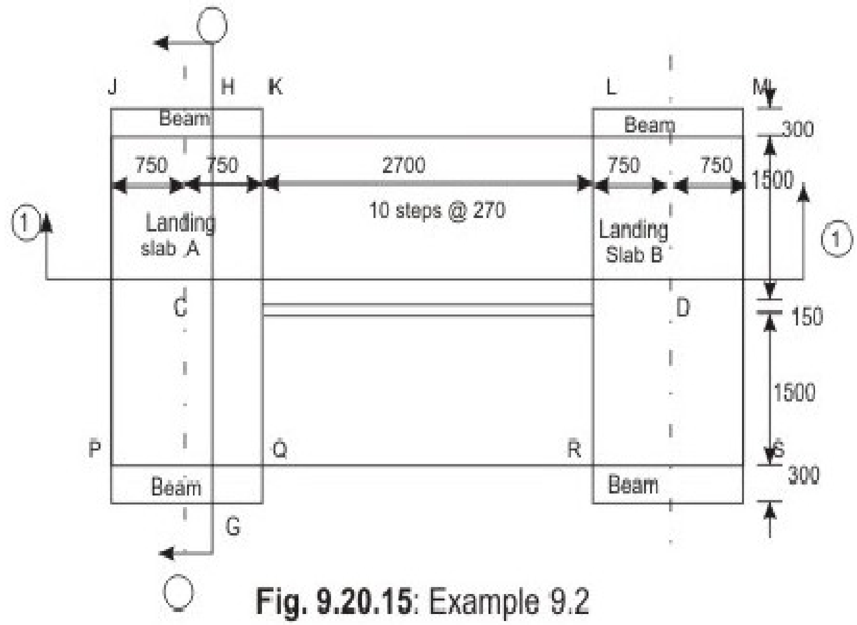


Fig. 9.20.15: Example 9.2

Design a trade-riser staircase shown in Fig.9.20.15 spanning longitudinally. Landing slabs are supported on beams spanning transversely. The dimensions of riser and trade are 160 mm and 270 mm, respectively. The finish loads and live loads are 1 kN/m^2 and 5 kN/m^2, respectively. Use M 20 and Fe 415.

Solution:

The distribution of loads on landings common to two spans perpendicular to each other shall be done as per cl. 33.2 of IS 456 (50% in each direction), since the going is supported on landing slabs which span transversely. The effective span in the longitudinal direction shall be taken as the distance between two centre lines of landings.

(A) Design of going

Step 1: Effective span and depth of slab

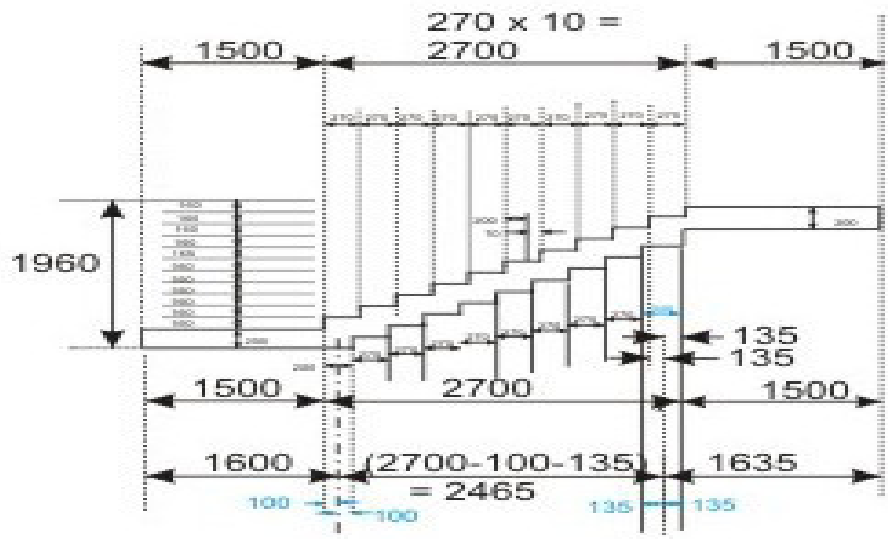


Fig. 9.20.16: Arrangement of loadings and Going of Example 9.2

Figure 9.20.16 shows the arrangement of the landings and going. The effective span is 4200 mm. Assume the thickness of trade-riser slab = 4200/25 = 168 mm, say 200 mm. The thickness of landing slab is also assumed as 200 mm.

Step 2: Calculation of loads (Fig. 9.20.17)

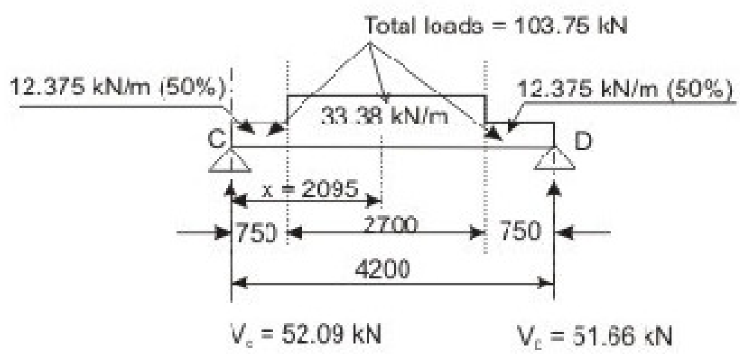


Fig. 9.20.17: Loads of section 1-1, Fig. 9.20.15, (Example 9.2)

The total loads including self-weight, finish and live loads on projected area of going (1500 mm x 2465 mm) is first determined to estimate the total factored loads per metre run.

(i) Self-weight of going

(a) Nine units of (0.2)(0.36)(1.5) @ 25(9) = 24.3 kN

(b) One unit of (0.27)(0.36)(1.5) @ 25(1) = 3.645 kN
(c) Nine units of (0.07)(0.2)(1.5) @ 25(9) = 4.725 kN
(ii) Finish loads @ 1 kN/m^2 = (1.5)(2.465)(1) = 3.6975 kN
(iii) Live loads @ 5 kN/m^2 = (1.5)(2.465)(5) = 18.4875 kN
Total = 54.855 kN
Factored loads per metre run = 1.5(54.855)/2.465 = 33.38 kN/m
(iv) Self-weight of landing slabs per metre run = 1.5(0.2)(25) = 7.5 kN/m
(v) Live loads on landings = (1.5)(5) = 7.5 kN/m
(vi) Finish loads on landings = (1.5)(1) = 1.5 kN/m
Total = 16.5 kN/m
Factored loads = 1.5(16.5) = 24.75 kN/m
Due to common area of landings only 50 per cent of this load should be considered. So, the loads = 12.375 kN/m. The loads are shown in Fig.9.20.17.

Step 3: Bending moment and shear force

Total factored loads = 33.38(2.465) + 12.375(0.85 + 0.885) = 103.75 kN
V_C = {12.375(0.85)(4.2 – 0.425) + 33.38(2.465)(0.885 + 1.2325)
+ 12.375(0.885)(0.885)(0.5)}/4.2 = 52.09 kN
V_D = 103.75 – 52.09 = 51.66 kN
The distance x from the left support where shear force is zero is now determined:
52.09 – 12.375(0.85) – 32.38(x – 0.85) = 0
or x = {52.09 – 12.375(0.85) + 33.38(0.85)}/33.38 = 2.095 m
Maximum factored bending moment at x = 2.095 m is
52.09(2.095) – 12.375(0.85)(2.095 – 0.425) – 33.38(2.095 – 0.85)(2.095 – 0.85)(0.5)
= 65.69 kNm

Step 4: Checking of depth of slab

From the maximum bending moment, we have
$d = \{(65690)/(1.5)(2.76)\}^{½}$ = 125.97 mm < 174 mm
From the shear force $V_u = V_A$, we get $v\tau$ = 52090/(1500)(174) = 0.199 N/mm^2. From cl. 40.2.1.1 and Table 19 of IS 456, we have $c\tau$ = (1.2)(0.28) = 0.336 N/mm^2. Table 20 of IS 456 gives max$c\tau$= 2.8 N/mm^2. Since, $v\tau$<$c\tau$<max$c\tau$, the depth of 200 mm is accepted.

Step 5: Determination of areas of steel reinforcement

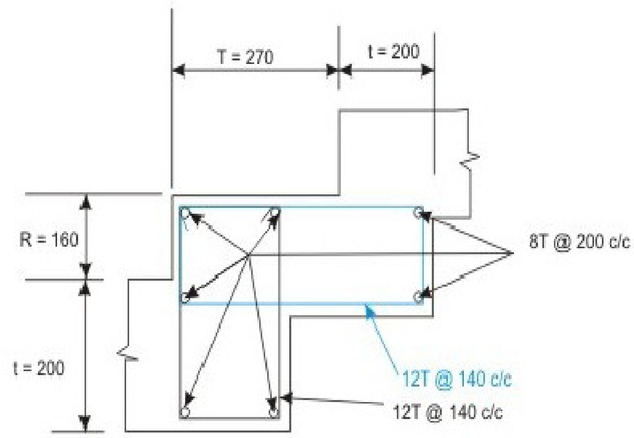


Fig. 9.20.18: Reinforcing bars - Example 9.2

$$M_u/bd^2 = 65.69(10^6)/(1500)(174)(174) = 1.446 \text{ N/mm}^2$$

Table 2 of SP-16 gives, p = 0.4416, to have A_{st} = 0.4416(1000)(174)/100 = 768.384 mm^2/m. Provide 12 mm diameter bars @ 140 mm c/c (= 808 mm^2) in form of closed ties (Fig.9.20.18).

Distribution bars: Area of distribution bars = 0.12(1000)(200)/100 = 240 mm^2/m.

Provide 8 mm diameter bars @ 200 mm c/c. The reinforcing bars are shown in Fig. 9.20.18.

(B) Design of landing slab

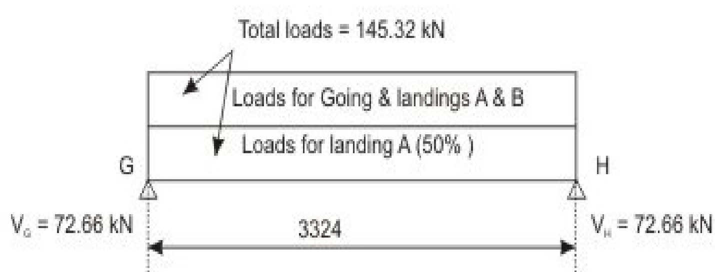


Fig. 9.20.19: Loads and reactions (Example 9.2)

Step 1: Effective span and depth of slab

With total depth D = 200 mm and effective depth d = 174 mm, the effective span (cl. 22.2a) = lesser of (1500 + 150 + 1500 + 174) and (1500 + 150 + 1500 + 300) = 3324 mm.

Step 2: Calculation of loads (Fig.9.20.19)

(i) Factored load of landing slab A = 50% of Step 2 (iv to vi) @ 12.375 kN/m = 12.375(3.324 = 41.1345 kN
(ii) Factored reaction V_C from one flight (see Step 3) = 52.09 kN

(iii) Factored reaction V_C from other flight = 52.09 kN

Total factored load = 145.32 kN. Due to symmetry of loads, $V_G = V_H$ = 72.66 kN. The bending moment is maximum at the centre line of GH.

Step 3: Bending moment and shear force (width *b* = 1500 mm)

Maximum bending moment = 145.32(3.324)/8 = 60.38 kNm
Maximum shear force $V_G = V_H$ = 145.32/2 = 72.66 kN

Step 4: Checking of depth of slab

From bending moment: $d = \{60380/(1.5)(2.76)\}^{½}$ = 120.77 mm < 174 mm. Hence o.k.

From shear force: $v\tau$ = 72660/(1500)(174) = 0.278 N/mm^2

From Step 4 of A: $c\tau$ = 0.336 N/mm^2, max$c\tau$= 2.8 N/mm^2. Hence, the depth is o.k. for shear also.

Step 5: Determination of areas of steel reinforcement

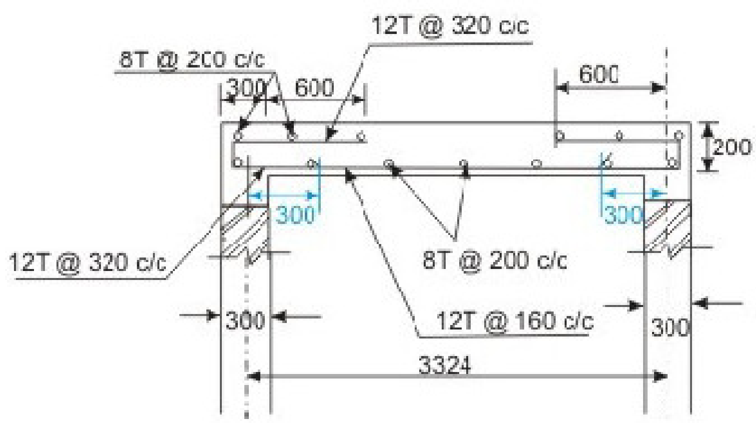


Fig. 9.20.20: Reinforcing bars of Example 9.2

M_u/bd^2 = 60380/(1.5)(174)(174) = 1.33 N/mm^2

Table 2 of SP-16 gives, p = 0.4022. So, A_{st} = 0.4022(1000)(174)/100 = 699.828 mm^2. Provide 12 mm diameter bars @ 160 mm c/c (= 707 mm^2).

Distribution steel area = (0.12/100)(1000)(200) = 240 mm^2

Provide 8 mm diameter @ 200 mm c/c (= 250 mm^2).

Step 6: Checking of development length

The moment M_1 for 12 mm @ 160 mm c/c (707 mm^2) = (707/699.828)60.38 = 60.998 kNm.

The shear force V = 72.66 kN. The diameter of the bar should be less/equal to {(1.3)(60.998)(10^6)/72.66(10^3)}/47 = 23.2 mm. Hence 12 mm diameter bars are o.k.
Use L_d = 47(12) = 564 mm, = 600 mm (say). The reinforcing bars are shown in Fig. 9.20.20.

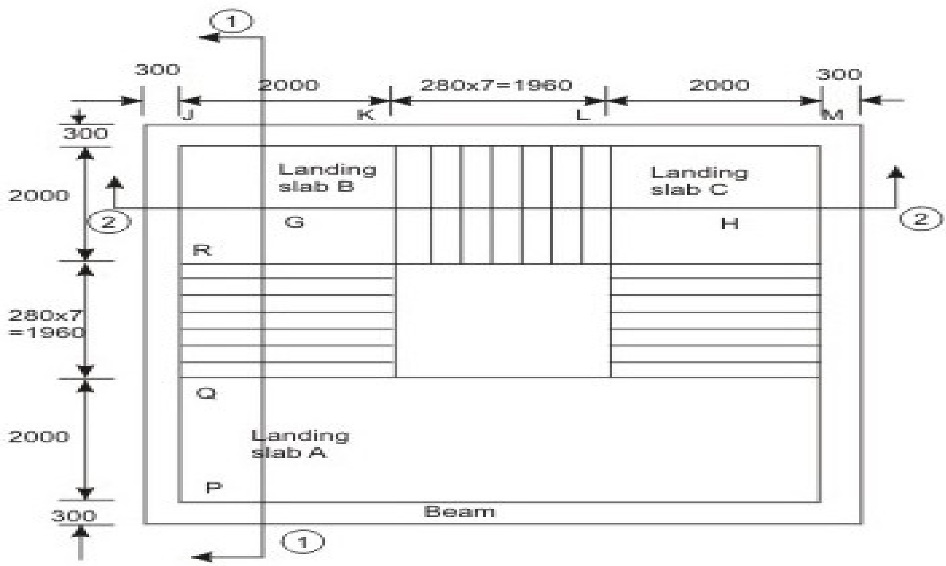


Fig. 9.20.21: Example Q.8

Design the open-well staircase of Fig.9.20.21. The dimensions of risers and trades are 160 mm and 270 mm, respectively. The finish loads and live loads are 1 kN/m^2 and 5 kN/m^2, respectively. Landing A has a beam at the edge while other landings (B and C) have brick walls. Use concrete of grade M 20 and steel of grade Fe 415.

Solution:

In this case landing slab A is spanning longitudinally along sec. 11 of Fig.9.20.21. Landing slab B is common to spans of sec. 11 and sec. 22, crossing at right angles. Distribution of loads on landing slab B shall be made 50 per cent in each direction (cl. 33.2 of IS 456). The effective span for sec. 11 shall be from the centre line of edge beam to centre line of brick wall, while the effective span for sec. 22 shall be from the centre line of landing slab B to centre line of landing slab C (cl. 33.1b of IS 456).

(A) Design of landing slab A and going (sec. 11 of Fig.9.20.21)

Step 1: Effective span and depth of slab

The effective span = 150 + 2000 + 1960 + 1000 = 5110 mm. The depth of waist slab is assumed as 5110/20 = 255.5 mm, say 250 mm. The effective depth = 250 – 20 – 6 = 224 mm. The landing slab is also assumed to have a total depth of 250 mm and effective depth of 224 mm.

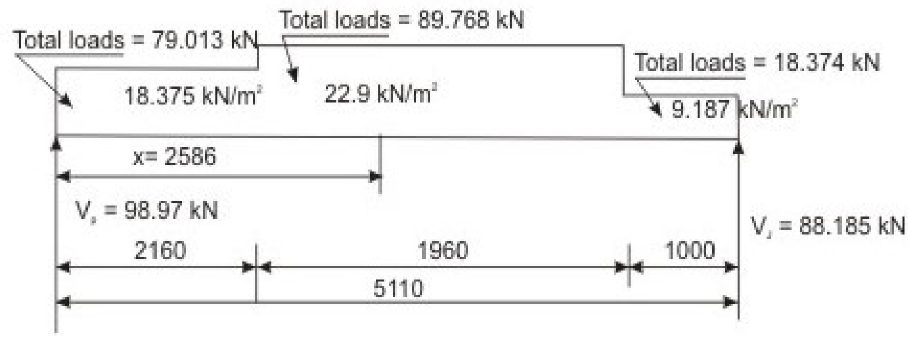


Fig 9.20.22: Calculation of loads sec 1-1 of Fig. 9.20.21, Example 9.8

Step 2: Calculation of loads (Fig.9.20.22)

(i) Loads on going (on projected plan area)

(a) Self weight of waist slab = 25(0.25)(313.85/270) = 7.265 kN/m^2

(b) Self weight of steps = 25(0.5)(0.16) = 2.0 kN/m^2

(c) Finish loads (given) = 1.0 kN/m^2

(d) Live loads (given) = 5.0 kN/m^2

Total = 15.265 kN/m^2

So, the factored loads = 1.5(15.265) = 22.9 kN/m^2

(ii) Landing slab A

(a) Self weight of slab = 25(0.25) = 6.25 kN/m^2

(b) Finish loads = 1.00 kN/m^2

(c) Live loads = 5.00 kN/m^2

Total = 12.25 kN/m^2

Factored loads = 1.5(12.25) = 18.375 kN/m^2

Landing slab B = 50 per cent of loads of landing slab A = 9.187 kN/m^2

The total loads of (i), (ii) and (iii) are shown in Fig.9.22.

Total loads (i) going = 22.9(1.96)(2) = 89.768 kN

Total loads (ii) landing slab A = 18.375(2.15)(2) = 79.013 kN

Total loads (iii) landing slab B = 9.187(1.0)(2) = 18.374 kN

Total loads = 187.155 kN

The loads are shown in Fig. 9.20.22.

Step 3: Bending moment and shear force (width = 2.0 m, Fig. 9.20.22)

V_P = {79.013(5.11 – 1.075) + 89.768(5.11 – 3.13) + 18.374(0.5)}/5.11
= 98.97 kN
V_J = 187.155 – 98.97 = 88.185 kN
The distance *x* where the shear force is zero is obtained from:
98.97 – 79.013 – 22.9(2)(*x* – 2.15) = 0
or *x* = 2.15 + (98.97 – 79.013)/22.9(2) = 2.586 m
Maximum bending moment at *x* = 2.586 m (width = 2 m)
= 98.97(2.586) – 79.013 – (22.9)(2)(0.436)(0.436)(0.5) = 161.013 kNm
Maximum shear force = 98.97 kN

Step 4: Checking of depth

From the maximum moment $d = \{161.013(10^3)/2(2.76)\}^{½}$ = 170.8 mm < 224 mm. Hence o.k.
From the maximum shear force, $v\tau$ = 98970/2000(224) = 0.221 N/mm^2. For the depth of slab as 250 mm, *k* = 1.1(cl. 40.2.1.1 of IS 456) and $c\tau$ = 1.1(0.28) = 0.308 N/mm^2 (Table 19 of IS 456). max$c\tau$ = 2.8 N/mm^2 (Table 20 of IS 456). Since, $v\tau < c\tau <$ max$c\tau$, the depth of slab as 250 mm is safe.

Step 5: Determination of areas of steel reinforcement

M_u/bd^2 = 161.013(10^3)/2(224)(224) = 1.60 N/mm^2. Table 2 of SP-16 gives *p* = 0.494, to have A_{st} = 0.494(1000)(224)/100 = 1106.56 mm^2/m. Provide 12 mm diameter bars @ 100 mm c/c (= 1131 mm^2/m) both for landings and waist slab.
Distribution reinforcement = 0.12(1000)(250)/100 = 300 mm^2/m. Provide 8 mm diameter @ 160 mm c/c (= 314 mm^2).

Step 6: Checking of development length

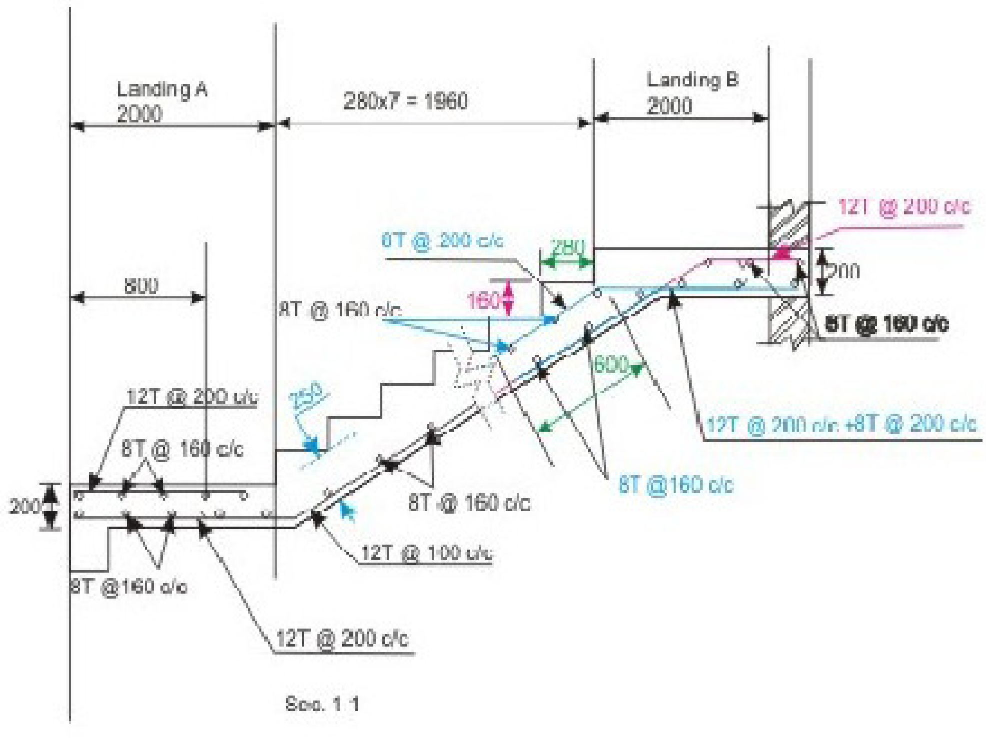


Fig. 9.20.23: Reinforcing bars, sec 1-1 of Fig 9.20.21, Example Q.8

Development length of 12 mm diameter bars 7(12) = 564 mm. Provide L_d = 600 mm.

For the slabs M_1 for 12 mm diameter @ 100 mm c/c = (1131)(161.013)/1106.56 = 164.57 kNm. Shear force = 98.97 kN. Hence, 47 φ ≤ 1.3(164.57)/98.97 ≤ 2161.67 mm or the diameter of main bar φ 45.99 mm. Hence, 12 mm diameter is o.k. The reinforcing bars are shown in Fig.9.20.23. ≤

(B) Design of landing slabs B and C and going (sec. 22 of Fig.9.20.21)

Step 1: Effective span and depth of slab

The effective span from the centre line of landing slab B to the centre line of landing slab C = 1000 + 1960 + 1000 = 3960 mm. The depths of waist slab and landing slabs are maintained as 250 mm like those of sec. 11.

Step 2: Calculation of loads (Fig.9.20.24)

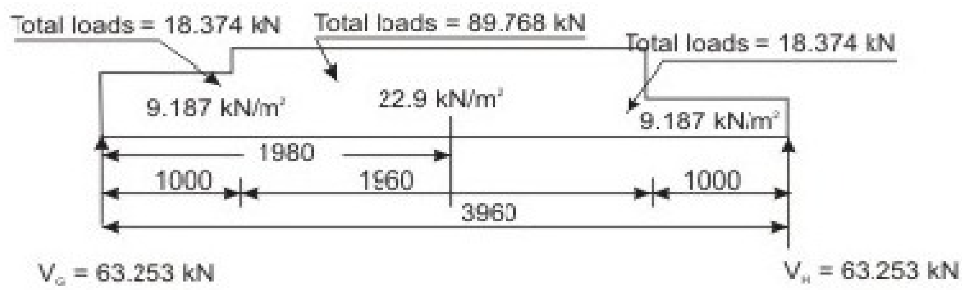


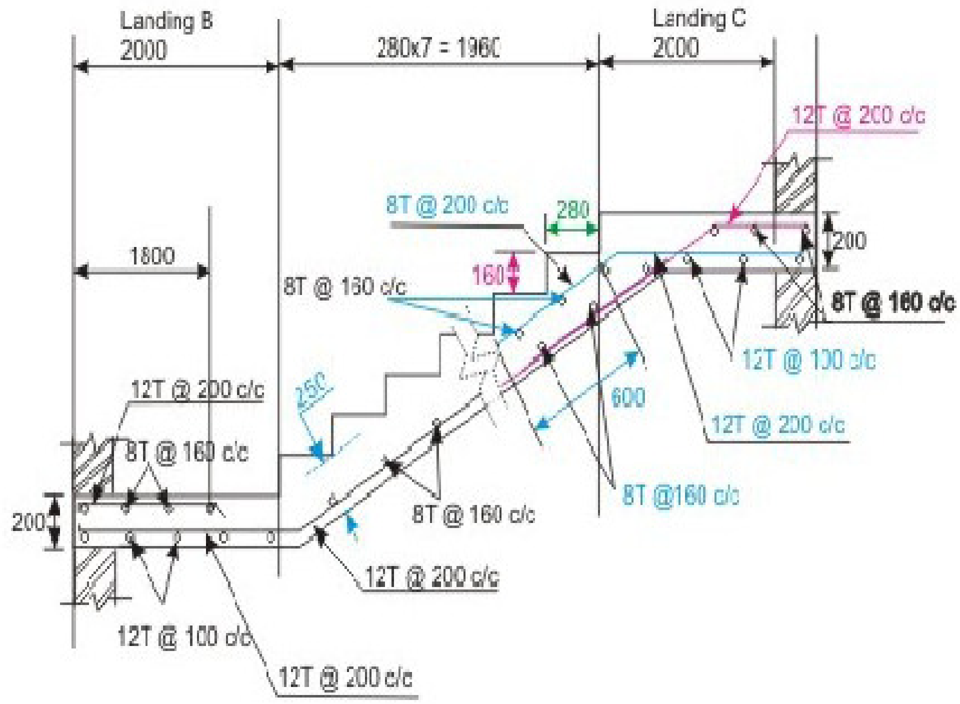
Fig 9.20.24: Calculation of loads, sec 2-2 of Fig. 9.20.21, Example Q.8

(i) Loads on going (Step 2(i) of A) = 22.9 kN/m^2
(ii) Loads on landing slab B (Step 2(iii)) = 9.187 kN/m^2
(iii) Loads on landing slab C (Step 2(iii)) = 9.187 kN/m^2
Total factored loads are:
(i) Going = 22.9(1.96)(2) = 89.768 kN
(ii) Landing slab A = 9.187(1.0)(2) = 18.374 kN
(iii) Landing slab B = 9.187(1.0)(2) = 18.374 kN
Total = 126.506 kN
The loads are shown in Fig.9.20.24.

Step 3: Bending moment and shear force (width = 2.0 m, Fig.9.20.24)

The total load is 126.506 kN and symmetrically placed to give $V_G = V_H$ = 63.253 kN. The maximum bending moment at x = 1.98 m (centre line of the span 3.96 m = 63.253(1.98) – 18.374(1.98 – 0.5) – 22.9(2)(0.98)(0.98)(0.5) = 76.05 kNm. Maximum shear force = 63.253 kN.
Since the maximum bending moment and shear force are less than those of the other section (maximum moment = 161.013 kNm and maximum shear force = 98.97 kN), the depth of 250 mm here is o.k. Accordingly, the amount of reinforcing bars are determined.

Step 4: Determination of areas of steel reinforcement



Sec. 2 2

Fig. 9.20.25: Reinforcing bars, sec 2-2 of Fig 9.20.21, Example Q.8

M_u/bd^2 = 76.05(10^3)/2(224)(224) = 0.76 N/mm^2. Table 2 of SP-16 gives p = 0.221. The area of steel = (0.221)(1000)(224)/100 = 495.04 mm^2. Providing 12 mm diameter @ 220 mm c/c gives 514 mm^2, however let us provide 12 mm diameter @ 200 mm c/c (565 mm^2) as it is easy to detail with 12 mm diameter @ 100 mm c/c for the other section. Distribution bars are same as for sec. 11 i.e., 8 mm diameter @ 160 mm c/c.

Step 5: Checking of development length

For the slab reinforcement 12 mm dia. @ 200 mm c/c, M_1 = (565)(76.05)/495.04 = 86.80 kNm, V = 63.25 kN. So, the diameter of main bar φ ≤ {(1.3)(86.80)(10^3)/(63.25)}/47, i.e., ≤ 37.96 mm. Hence, 12 mm diameter bars are o.k. Distribution steel shall remain the same as in sec. 11, i.e., 8 mm diameter @ 160 mm c/c.

The reinforcing bars are shown in Fig.9.20.25. Figures 9.20.23 and 9.20.25 show the reinforcing bars considered separately. However, it is worth mentioning that the common areas (landing B and C) will have the bars of larger areas of either section eliminating the lower bars of other section.

EX- 2

Design the staircase of illustrative example 9.1 of Fig.9.20.10 if supported on beams along KQ and LR only making both the landings A and B as cantilevers. Use the finish loads = 1 kN/m^2, live loads = 5 kn/m^2, riser R = 160 mm, trade T = 270 mm, grade of concrete = M 20 and grade of steel = Fe 415.

(25 marks)

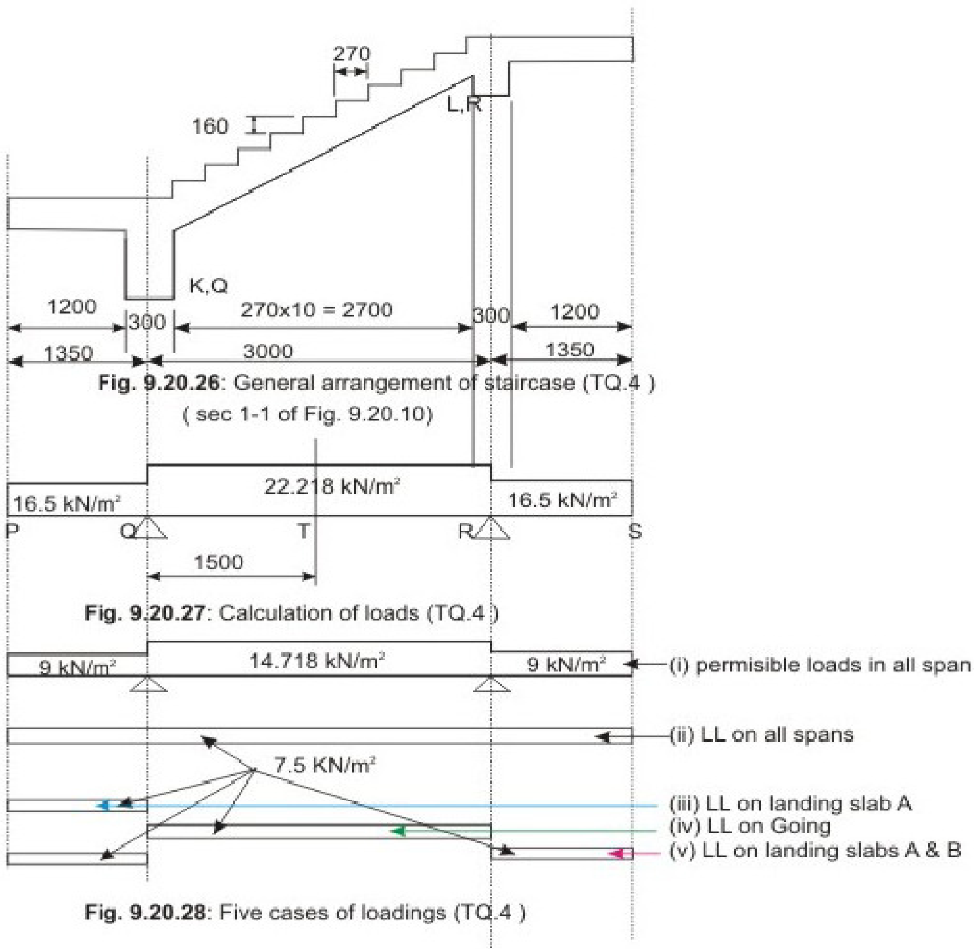


Fig. 9.20.26: General arrangement of staircase (TQ.4) (sec 1-1 of Fig. 9.20.10)

Fig. 9.20.27: Calculation of loads (TQ.4)

Fig. 9.20.28: Five cases of loadings (TQ.4)

Solution:

The general arrangement is shown in Fig.9.20.26. With R = 160 mm and T = 270 mm, the inclined length of each step = $\{(160)^2 + (270)^2\}^{½}$ = 313.85 mm. The structural arrangement is that the going is supported from beams along KQ and LR and the landings A and B are cantilevers.

Step 1: Effective span and depth of slab

As per cl. 33.1a of IS 456, the effective span of going = 3000 mm and as per cl. 22.2c of IS 456, the effective length of cantilever landing slabs = 1350 mm. The depth of waist slab and landing is kept at 200 mm (greater of 3000/20 and 1350/7). The effective depth = 200 – 20 – 6 = 174 mm.

Step 2: Calculation of loads

(i) Loads on going (on projected plan area)

(a) Self weight of waist-slab = 25(0.20)(313.85)/270 = 6.812 kN/m^2

(b) Self weight of steps = 25(0.5)(0.16) = 2.0 kN/m^2

(c) Finishes (given) = 1.0 kN/m^2

(d) Live loads (given) = 5.0 kN/m^2

Total: 14.812 kN/m^2

Total factored loads = 1.5(14.812) = 22.218 kN/m^2

(ii) Loads on landing slabs A and B

(a) Self weight of landing slabs = 25(0.2) = 5 kN/m^2

(b) Finishes (given) = 1 kN/m^2

(c) Live loads (given) = 5 kN/m^2

Total: 11 kN/m^2

Total factored loads = 1.5(11) = 16.5 kN/m^2. The total loads are shown in Fig.9.20.27.

Step 3: Bending moments and shear forces

Here, there are two types of loads: (i) permanent loads consisting of self-weights of slabs and finishes for landings and self-weights of slab, finishes and steps for going, and (ii) live loads. While the permanent loads will be acting everywhere all the time, the live loads can have several cases. Accordingly, five different cases are listed below. The design moments and shear forces will be considered taking into account of the values in each of the cases,. The different cases are (Fig.9.20.28):

(i) Permanent loads on going and landing slabs
(ii) Live loads on going and landing slabs
(iii) Live loads on landing slab A only
(iv) Live loads on going only
(v) Live loads on landing slabs A and B only

The results of V_Q, V_R, negative bending moment at Q and positive bending moment at T (Fig.9.20.27) are summarized in Table 9.2. It is seen from Table 9.2 that the design moments and shear forces are as follows:

(a) Positive bending moment = 25.195 kNm at T for load cases (i) and (iv).
(b) Negative bending moment = -22.553 kNm at Q for load cases (i) and (ii).
(c) Maximum shear force = 83.4 kN at Q and R for load cases (i) and (ii).

Table 9.2 Values of reaction forces and bending moments for different cases of loadings (Example: TQ.4, Figs. 9.20.26 to 9.20.28)

Case	V_Q (kN)	V_R (kN)	Negative moment at Q (kNm)	Positive moment at T (kNm)
Permanent loads on going and landings	+51.34	+51.34	-12.302	+12.535
Live loads on going and landings	+32.06	+32.06	-10.251	+2.404
Live loads on landing A only	+18.605	-3.417	-10.251	-5.13
Live loads on	+16.875	+16.875	0	+2.66

DESIGN OF FLAT SLAB

Introduction

Common practice of design and construction is to support the slabs by beams and support the beams by columns. This may be called as beam-slab construction. The beams reduce the available net clear ceiling height. Hence in warehouses, offices and public halls some times beams are avoided and slabs are directly supported by columns. These types of construction are aesthetically appealing also. These slabs which are directly supported by columns are called Flat Slabs. Fig. 1.1 shows a typical flat slab.

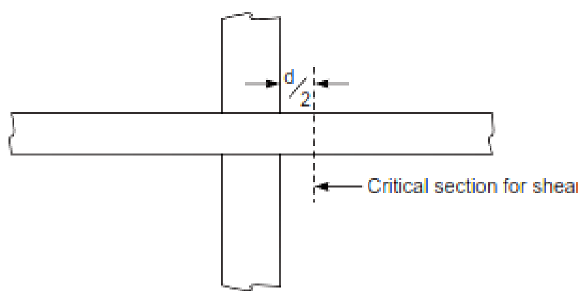


Fig. 1.1 *A typical flat slab (without drop and column head)*

The column head is some times widened so as to reduce the punching shear in the slab. The widened portions are called column heads. The column heads may be provided with any angle from the consideration of architecture but for the design, concrete in the portion at 45º on either side of vertical only is considered as effective for the design [Ref. Fig. 1.2].

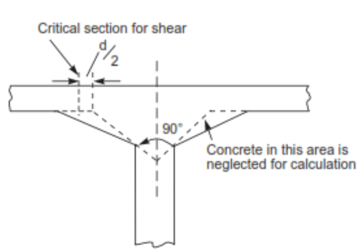


Fig. 1.2 *Slab without drop and column with column head*

PRINCIPLES OF DESIGN OF MAT FOUNDATION

Foundations

A foundation is defined as the engineered interface between the earth and the structure it supports that transmits the loads to the soil or rock. The design differs from structural design in that the choices in material and framing system are not available, and quality of materials cannot be assured. Foundation design is dependent on geology and climate of the site.

Soil Mechanics

Soil is another building material and the properties, just like the ones necessary for steel and concrete and wood, must be known before designing. In addition, soil has other properties due to massing of the material, how soil particles pack or slide against each other, and how water affects the behavior. The important properties are

- specific weight (density)
- allowable soil pressure
- factored net soil pressure – allowable soil pressure less surcharge with a factor of safety
- shear resistance
- backfill pressure
- cohesion & friction of soil
- effect of water
- settlement
- rock fracture behavior

Structural Strength and Serviceability

There are significant serviceability considerations with soil. Soils can settle considerably under foundation loads, which can lead to redistribution of moments in continuous slabs or beams, increases in stresses and cracking. Excessive loads can cause the soil to fail in bearing and in shear. The presence of water can cause soils to swell or shrink and freeze and thaw, which causes heaving. Fissures or fault lines can cause seismic instabilities. A geotechnical engineer or engineering service can use tests on soil bearings from the site to determine the ultimate bearing capacity, q_u. Allowable stress design is utilized for soils because of the variability do determine the allowable bearing capacity, $q_a = q_u/(\text{safety factor})$.

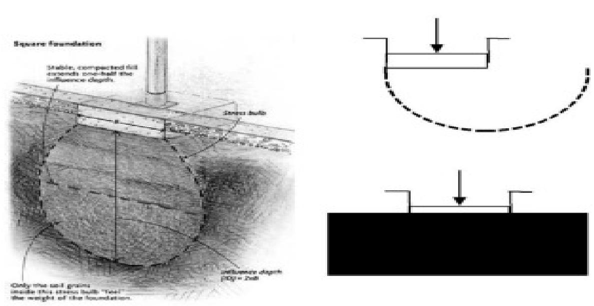


Fig 3.27 Strength and serviceability

Soil acts somewhat like water, in that it exerts a lateral pressure because of the weight of the material above it, but the relationship is not linear. Soil can have an active pressure from soil behind a retaining wall and passive pressure from soil in front of the footing. Active pressure is typically greater than passive pressure.

Foundation Materials

Typical foundation materials include:

- steel
- wood
- composites, i.e. steel tubing filled with concrete

Foundation Design

Table 7-1 Average Bearing Capacities of Various Foundation Beds

Soil	Bearing Capacity, q_a (ksf)
Alluvial soil	≤ 1
Soft clay	2
Firm clay	4
Wet sand	4
Sand and clay mixed	4
Fine dry sand (compact)	6
Hard clay	8
Coarse dry sand (compact)	8
Sand and gravel mixed (compact)	10
Gravel (compact)	12
Soft rock	16
Hard pan or hard shale	20
Medium rock	30
Hard rock	80

Design of foundations with variable conditions and variable types of foundation structures will be different, but there are steps that are typical to every design, including:

(i) Calculate loads from structure, surcharge, active & passive pressures, etc.
(ii) Characterize soil – hire a firm to conduct soil tests and produce a report that includes soil material properties
(iii) Determine footing location and depth – shallow footings are less expensive, but the variability of the soil from the geotechnical report will drive choices
(iv) Evaluate soil bearing capacity – the factor of safety is considered here
(v) Determine footing size – these calculations are based on working loads and the allowable soil pressure

Shallow Foundation Types

Considered simple and cost effective because little soil is removed or disturbed.

Spread footing –A single column bears on a square or Rectangular pad to distribute the load over a bigger area.

Wall footing – A continuous wall bears on a wide pad to distribute the load.
Eccentric footing – A spread or wall footing that also must resist amoment in addition to the axial column load.
Combined footing – Multiple columns (typically two) bear on arectangular or trapezoidal shaped footing.
Unsymmetrical footing – A footing with a shape that does not
Evenly distribute bearing pressure from column loads and moments. It typically involves a hole or a non-rectangular shape influenced by a boundary or property line.
Strap footing – A combined footing consisting of two spread footings with a beam or strap connecting the slabs. The purpose of this is to limit differential settlements.
Mat foundation – A slab that supports multiple columns. The mat can be stiffened with a grid or grade beams. It is typically used when the soil capacity is very low.

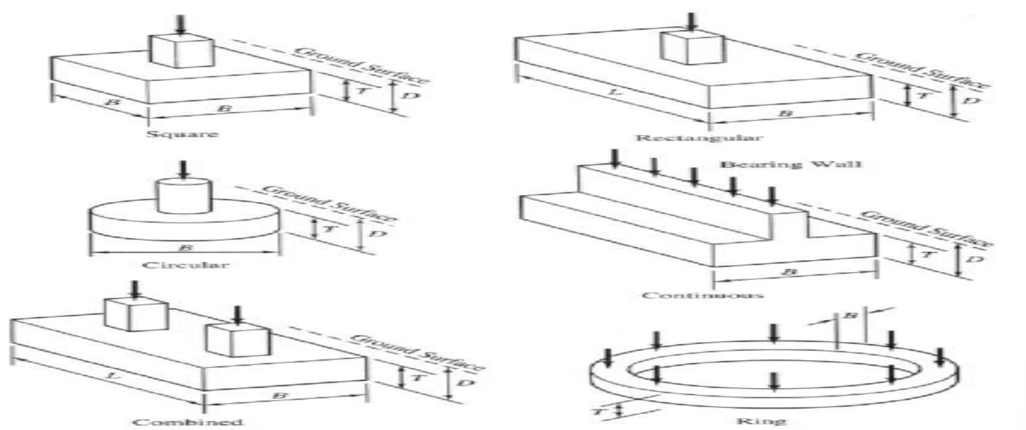


Figure 5.1 Spread footing shapes and dimensions.

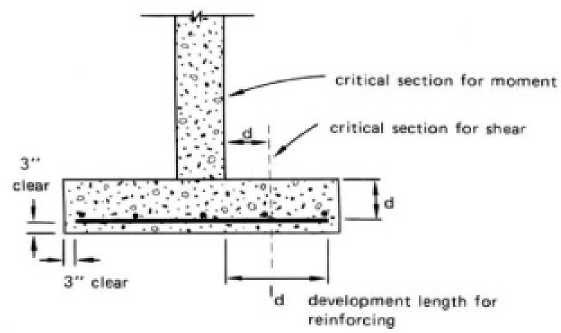
Deep Foundation Types

Considerable material and excavation is required, increasing cost and effort.
Retaining Walls –A wall that retains soil or other materials, and must resist sliding and overturning. Can have counter forts, buttresses or keys.
Basement Walls –A wall that encloses a basement space, typically next to a floor slab, and that may be restrained at the top by a floor slab.
Piles – Next choice when spread footings or mats won't work, piles are used to distribute loads by end bearing to strong soil or friction to low strength soils. Can be used to resist uplift, a moment causing overturning, or to compact soils. Also useful when used in combination to control settlements of mats

DESIGN

For the 16 in. thick 8.5 ft. square reinforced concrete footing carrying 150 kips dead load and 100 kips live load on a 24 in. square column, determine if the footing thickness is adequate for 4000 psi. A 3 in. cover is required with Concrete in contact with soil. Also determine the moment for reinforced concrete design.

SOLUTION:



1. Find design soil pressure: $q_u = \frac{P_u}{A}$

P_u = 1.2D + 1.6L = 1.2 (150 k) + 1.6 (100 k) = 340 k

$q_u = \frac{340k}{(8.5\ ft)^2}$ = 4.71 k/ft^2

2. Evaluate one-way shear at d away from column face (Is $V_u < \phi V_c$?)

d = h_f – c.c. – distance to bar intersection

presuming #8 bars:

d = 16 in. – 3 in. (soil exposure) - 1 in. x (1 layer of #8's) = 12 in.

V_u = total shear = q_u (edge area)

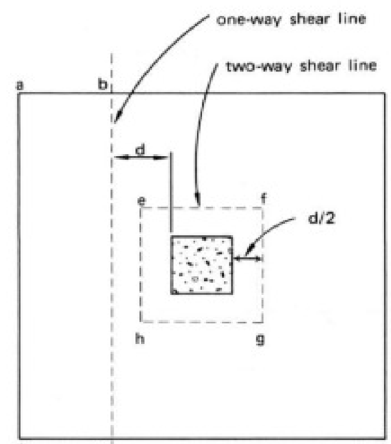
V_u on a 1 ft strip = q_u (edge distance) (1 ft)

V_u = 4.71 k/ft^2 [(8.5 ft – 2 ft)/2 – (12 in.)(1 ft/12 in.)] (1 ft) = 10.6 k

ϕV_c = one-way shear resistance = $\phi 2 \sqrt{f_c'}$ bd

for a one foot strip, b = 12 in.

ϕV_c = 0.75(2 $\sqrt{4000}$ psi)(12 in.)(12 in.) = 13.7 k > 10.6 k OK



3. Evaluate two-way shear at d/2 away from column face (Is $V_u < \phi V_c$?) b_o = perimeter = 4 (24 in. + 12 in.) = 4 (36 in.) = 144 in

Vu = total shear on area outside perimeter = Pu – qu (punch area)

V_u = 340 k – (4.71 k/ft^2)(36 in.)^2(1 ft/12 in.)^2 = 297.6 kips

ϕV_c = two-way shear resistance = $\phi 4 \sqrt{f_c'} b_o d$ = 0.75(4$\sqrt{4000}$ psi)(144 in.)(12 in.) = 327.9 k > 297.6 k OK

4. Design for bending at column face

$M_u = w_u L^2/2$ for a cantilever. L = (8.5 ft – 2 ft)/2 = 3.25 ft, and w_u for a 1 ft strip = q_u (1 ft) M_u = 4.71 k/ft^2(1 ft)(3.25 ft)^2/2 = 24.9 k-ft (per ft of width)

To complete the reinforcement design, use b =12 in. and trial d = 12 in., choose ρ, determine A_s, find if $\phi M_n > M_u$... ϕV_c = 0.75(2)

Introduction to prestressed concrete-Principles
Introduction

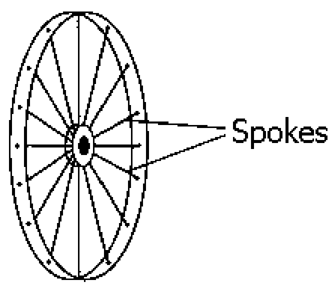
The development of prestressed concrete can be studied in the perspective of traditional building materials. In the ancient period, stones and bricks were extensively used. These materials are strong in compression, but weak in tension. For tension, bamboos and coir ropes were used in bridges. Subsequently iron and steel bars were used to resist tension.

These members tend to buckle under compression. Wood and structural steel members were effective both in tension and compression.

In reinforced concrete, concrete and steel are combined such that concrete resists compression and steel resists tension. This is a passive combination of the two materials.
In prestressed concrete high strength concrete and high strength steel are combined such that the full section is effective in resisting tension and compression. This is an active combination of the two materials. The following sketch shows the use of the different materials with the progress of time.

Pre-tensioning the spokes in a bicycle wheel

The pre-tension of a spoke in a bicycle wheel is applied to such an extent that there will Always is a residual tension in the spoke.



Pre-tensioning the spokes in a bicycle wheel

For concrete, internal stresses are induced (usually, by means of tensioned steel) for the following reasons.

- The tensile strength of concrete is only about 8% to 14% of its compressive strength.
- Cracks tend to develop at early stages of loading in flexural members such as beams and slabs.
- To prevent such cracks, compressive force can be suitably applied in the perpendicular direction.
- Prestressing enhances the bending, shear and torsional capacities of the flexural members.
- In pipes and liquid storage tanks, the hoop tensile stresses can be effectively counteracted by circular prestressing.

Prestressed concrete Applications

Prestressed concrete is the main material for floors in high-rise buildings and the entire containment vessels of nuclear reactors.

Unbounded post-tensioning tendons are commonly used in parking garages as barrier cable.[3] Also, due to its ability to be stressed and then de-stressed, it can be used to temporarily repair a damaged building by holding up a damaged wall or floor until permanent repairs can be made.

The advantages of prestressed concrete include crack control and lower construction costs; thinner slabs - especially important in high rise buildings in which floor thickness savings can translate into additional floors for the same (or lower) cost and fewer joints, since the distance that can be spanned by post-tensioned slabs exceeds that of reinforced constructions with the same thickness. Increasing span lengths increases the usable unencumbered floor space in buildings; diminishing the number of joints leads to lower maintenance costs over the design life of a building, since joints are the major focus of weakness in concrete buildings.

The first prestressed concrete bridge in North America was the Walnut Lane Memorial Bridge in Philadelphia, Pennsylvania. It was completed and opened to traffic in 1951. Prestressing can also be accomplished on circular concrete pipes used for water transmission. High tensile strength steel wire is helically-wrapped around the outside of the pipe under controlled tension and spacing which induces a circumferential compressive stress in the core concrete. This enables the pipe to handle high internal pressures and the effects of external earth and traffic loads.

Introduction

The prestressing and precasting of concrete are inter-related features of the modern building industry. Through the application of imaginative design and quality control, they have, since the 1930's, had an increasing impact on architectural and construction procedures. Prestressing of concrete is the application of a compressive force to concrete members and may be achieved by either pretensioning high tensile steel strands before the concrete has set, or by post-tensioning the strands after the concrete has set. Although these techniques are commonplace, misunderstanding of the principles, and the way they are applied, still exists. This paper is aimed at providing a clear outline of the basic factors differentiating each technique and has been prepared to encourage understanding amongst those seeking to broaden their knowledge of structural systems.

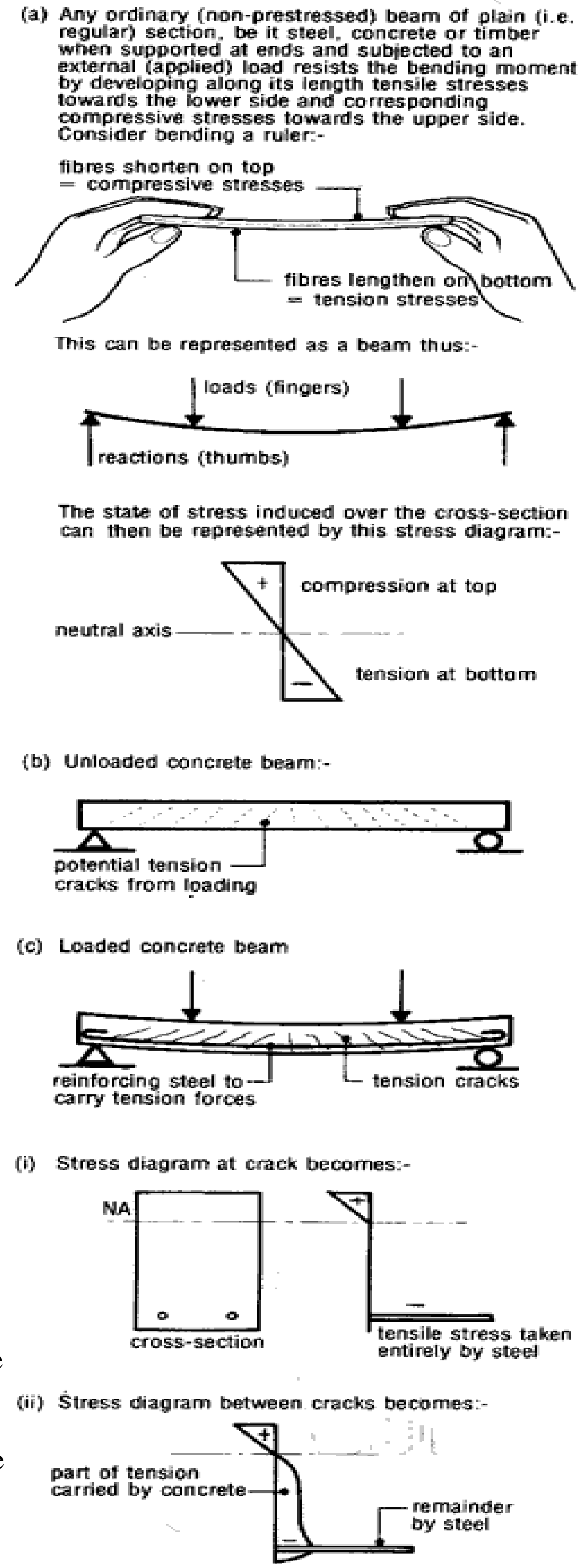
2. Definitions

2.1 Prestressed Concrete

Prestressing of concrete is defined as the application of compressive stresses to concrete members. Those zones of the member ultimately required to carry tensile stresses under working load conditions are given an initial compressive stress before the application of working loads so that the tensile stresses developed by these working loads are balanced by induced compressive strength. Prestress can be applied in two ways - Pre-tensioning or Post-tensioning.

2.2 Pre-tensioning

Pre-tensioning is the application, before casting, of a tensile force to high tensile steel tendons around which the concrete is to be cast. When the placed concrete has developed sufficient compressive strength a compressive force is imparted to it by releasing the tendons, so that the concrete member is in a permanent state of prestress.



2.3 Post-tensioning

Post-tensioning is the application of a compressive force to the concrete at some point in time after casting. When the concrete has gained strength a state of prestress is induced by tensioning steel tendons passed through ducts cast into the concrete, and locking the stressed tendons with mechanical anchors. The tendons are then normally grouted in place.

3.Advanages of Prestressing

3.1 General Advantages

The use of prestressed concrete offers distinct advantages over ordinary reinforced concrete. These advantages can be briefly listed as follows:

1. Prestressing minimises the effect of cracks in concrete elements by holding the concrete in compression.
2. Prestressing allows reduced beam depths to be achieved for equivalent design strengths.
3. Prestressed concrete is resilient and will recover from the effects of a greater degree of overload than any other structural material.
4. If the member is subject to overload, cracks, which may develop, will close up on removal of the overload.
5. Prestressing enables both entire structural elements and structures to be formed from a number of precast units, e.g. Segmented and Modular Construction.
6. Lighter elements permit the use of longer spanning members with a high strength to weight characteristic.
7. The ability to control deflections in prestressed beams and slabs permits longer spans to be achieved.
8. Prestressing permits a more efficient usage of steel and enables the economic use of high tensile steels and high strength concrete.

3.2 Cost advantages of Prestressing

Prestressed sconcrete can provide significant cost advantages over structural steel sections or ordinary reinforced concrete.

4. Limitations of Prestressing

The limitations of prestressed concrete are few and really depend only upon the imagination of the designer and the terms of his brief. The only real limitation where prestressing is a possible solution may be the cost of providing moulds for runs of limited quantity of small numbers of non-standard units.

5. Fundamentals of Prestressing

5.1 The Tensile Strength of Concrete

The tensile strength of unreinforced concrete is equal to about 10% of its compressive strength. Reinforced concrete design has in the past neglected the tensile strength of unreinforced concrete as being too unreliable. Cracks in the unreinforced concrete occur for many reasons and destroy the tensile capability. See Fig.1.With prestressed concrete design however, the tensile strength of concrete is not neglected. In certain applications it is used as part of the design for service loadings. In ordinary reinforced concrete, steel bars are introduced to overcome this low tensile strength. They resist tensile forces and limit the width of cracks that will develop under design loadings. Reinforced concrete is thus designed assuming the concrete to be cracked and unable to carry any tensile force. Prestressing gives crack-free construction by placing the concrete in compression before the application of service loads.

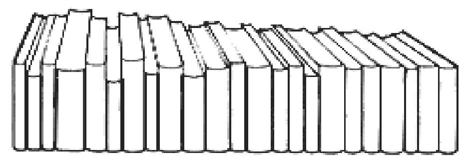


FIG. 2(A)

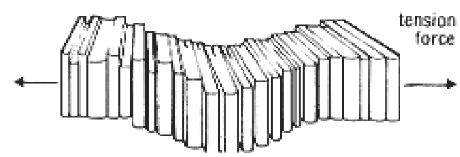


FIG. 2(B)

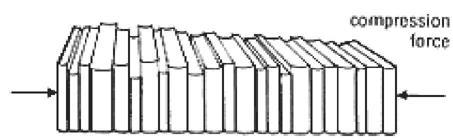


FIG. 2(C)

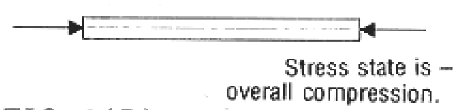


FIG. 2(D)

5.2 The Basic Idea

A simple analogy to prestressing will best explain the basic idea. Consider a row of books or blocks set up as a beam. See Fig.2(a). This "beam" is able to resist compression at the top but is unable to resist any tension forces at the bottom as the "beam" is now like a badly cracked concrete member, i.e. the discontinuity between the books ensures that the "beam" has no inherent tension resisting properties. If it is temporarily supported and a tensile force is applied, the "beam'' will fail by the books dropping out along the discontinuities. See Fig.2(b). For the beam then to function properly a compression force must be applied as in Fig.2(c). The beam is then "prestressed" with forces acting in an opposite direction to those induced by loading. The effect of the longitudinal prestressing force is thus to produce pre-compression in the beam before external downward loads are applied. The application of the external downward load merely reduces the proportion of precompression acting in the tensile zone of the beam.

5.3 The Position of the Prestressing Force

Prestressing can be used to best advantage by varying the position of the prestress force. When the prestress is applied on the centroid of the cross-section a uniform compression is obtained. Consider the stress state of the beam in Fig.3(b). We can see that by applying a prestress of the right magnitude we can produce pre-compression equal and opposite to the tensile force in Fig.3(b).Then by adding the stress blocks we get: i.e. zero stress towards the bottom fibres and twice the compressive stress towards the top fibres. Now apply the pre-compression force at 1/3 the beam depth above the bottom face. As well as the overall compression we now have a further compressive stress acting on the bottom fibre due to the moment of the eccentric prestress force about the neutral axis of the section. We then find it is possible to achieve the same compression at the bottom fibre with only half the prestressing force. See Fig.3(d). Adding now the stress blocks of Fig.3(b) and 3(d) we find that the tensile stress in the bottom fibre is again negated whilst the final compressive stress in the top fibre is only half that of Fig.3(c). See Fig.3(e).

Thus by varying the position of the compressive force we can reduce the prestress force required, reduce the concrete strength required and sometimes reduce the cross sectional area. Changes in cross sections such as using T or I or channel sections rather than rectangular sections can lead to further economies.

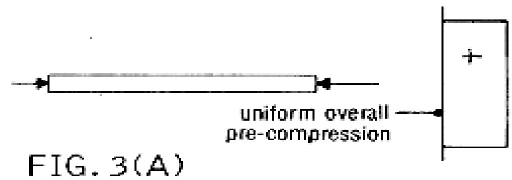


FIG. 3(A)

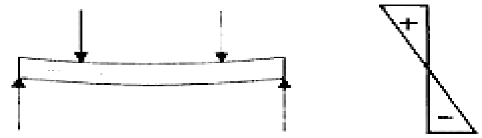


FIG. 3(B)

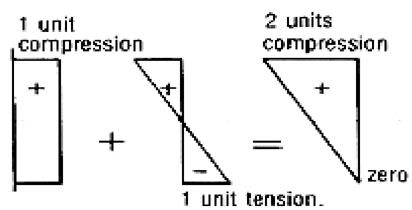


FIG. 3(C)

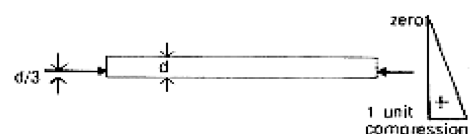


FIG. 3(D)

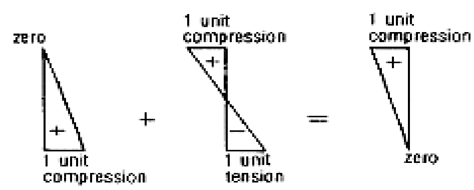


FIG. 3(E)

5.4 The Effect of Prestress on Beam Deflection

From 5.3 it is obvious that the designer should, unless there are special circumstances, choose the eccentrically applied prestress. Consider again the non-prestressed beam of Fig.1(a). Under external loads the beam deflects to a profile similar to that exaggerated in Fig.4(a). By applying prestress eccentrically a deflection is induced. When the prestress is applied in the lower portion of the beam, the deflection is upwards producing a hogging profile. See Fig.4(b). By applying the loads of Fig.4(a) to our prestressed beam, the final deflection shape produced is a sum of Figs.4(a) and 4(b) as shown in Fig.4(c). Residual hogging, though shown exaggerated in the Fig.4(c), is controlled within limits by design code and bylaw requirements. Such control of deflection is not possible with simple reinforced concrete. Reductions in deflections under working loads can then be achieved by suitable eccentric prestressing. In long span members this is the controlling factor in the choice of the construction concept an technique employed.

5.5 Prestress Losses

Most materials to varying degrees are subject to "creep", i.e. under a sustained permanent load the material tends to develop some small amount of plasticity and will not return completely to its original shape. There has been an irreversible deformation or permanent set. This is known as "creep" Shrinkage of concrete and "creep" of concrete and of steel reinforcement are potential sources of prestress loss and are provided for in the design of prestressed concrete. Shrinkage:The magnitude of shrinkage may be in the range of 0.02% depending on the environmental conditions and type of concrete.With pre-tensioning, shrinkage starts as soon as the concrete is poured whereas with post-tensioned concrete there is an opportunity for the member to experience part of its shrinkage prior to tensioning of the tendon, thus pre-compression loss from concrete shrinkage is less.

Creep: With prestressing of concrete the effect is to compress and shorten the concrete. This shortening must be added to that of concrete shrinkage. In the tensioned steel tendons the effect of "creep'' is to lengthen the tendon causing further stress loss. Allowance must be made in the design process for these losses. Various formulae are available.

Pull-in: With all prestressing systems employing wedge type gripping devices, some degree of pull-in at either or both ends of a pre-tensioning bed or post-tensioned member can be expected. In normal operation, for most devices in common use, this pull-in is between 3mm and 13mm and allowance is made when tensioning the tendons to accommodate this.

5.6 Materials

5.6.1 Steel

Early in the development of prestressing it was found that because of its low limit of elasticity ordinary reinforcing steel could not provide sufficient elongation to counter concrete shortening due to creep and shrinkage. it is necessary to use the high tensile steels which were developed to produce a large elongation when tensioned. This ensures that there is sufficient elongation reserve to maintain the desired pre-compression. The relationship between the amount of load, or stress, in a material and the stretch, or strain, which the material undergoes while it is being loaded is depicted by a stress-strain curve. At any given stress there is a corresponding strain. Strain is defined as the elongation of a member divided by the length of the member. The stress-strain properties of some grades of steel commonly encountered in construction are shown in Fig.5. It is apparent from these relationships that considerable variation exists between the properties of these steels. All grades of steel have one feature in common: the relationship between stress and strain is a straight line below a certain stress. The stress at which this relationship departs from the straight line is called the yield stress, and is denoted by the symbol fy in Fig. 5. A conversion factor may be used to convert either stress to strain or strain to stress in this range. This conversion factor is called the modulus of elasticity E. Structural grade steels which are commonly used for rolled structural sections and reinforcing bars, show a deviation from this linear relationship at a much lower

FIG. 4(A)

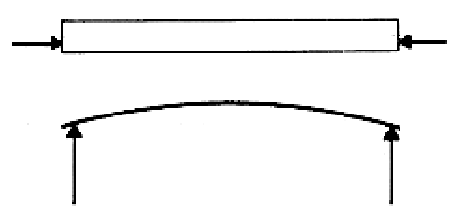


FIG. 4(B)

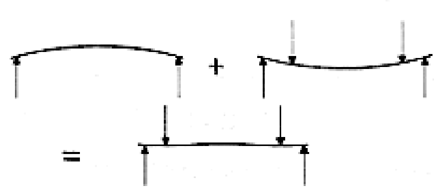


FIG. 4(C)

stress than high strength prestressing steel. High strength steels cannot be used for reinforced concrete as the width of cracks under loading would be unacceptably large. These high strength steels achieve their strength largely through the use of special compositions in conjunction with cold working. Smaller diameter wires gain strength by being cold drawn through a number of dies. The high strength of alloy bars is derived by the use of special alloys and some working.

5.6.2 Concrete

To accommodate the degree of compression imposed by the tensioning tendons and to minimise prestress losses, a high strength concrete with low shrinkage properties is required. Units employing high strength concrete are most successfully cast under controlled factory conditions.

6. Prestressing Methods

6.1 General

Methods of prestressing concrete fall into two broad categories differentiated by the stage at which the prestress is applied.That is, whether the steel is pre-tensioned or post-tensioned. From the definitions para 2.2 pre-tensioning is stated as "the application before casting, of a tensile force to high tensile steel tendons around which the concrete is cast. . ." and para 2.3 "Post-tensioning is the application of a compressive force to the concrete at some point in time after casting. When the concrete has hardened a state of prestress is induced by tensioning steel tendons passing through ducts cast into the Concrete".

6.2 Types of Tendon

There are three basic types of tendon used in the prestressing of concrete:

Bars of high strength alloy steel.These bar type tendons are used in certain types of post-tensioning systems. Bars up to 40mm diameter are used and the alloy steel from which they are made has a yield stress (fy Fig.5) in the order of 620 MPa. This gives bar tendons a lower strength to weight ratio than either wires or strands, but when employed with threaded anchorages has the advantages of eliminating the possibility of pull-in at the anchorages as discussed in para. 5.5, and of reducing anchorage costs.

Wire, mainly used in post-tensioning systems for prestressing concrete, is cold drawn and stress relieved with a yield stress of about 1300 MPa.

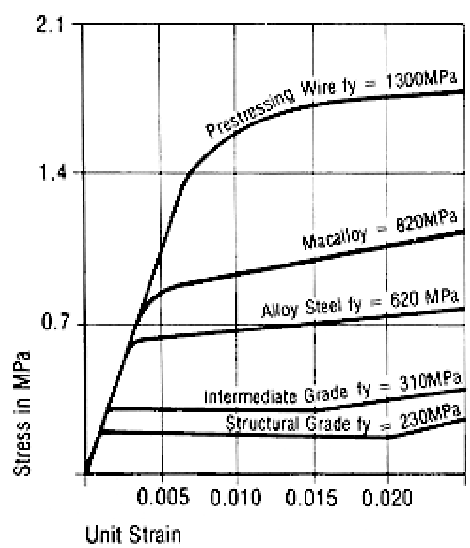


FIG. 5. STRESS-STRAIN

Wire diameters most commonly used in New Zealand are 5mm, 7mm, and 8mm.

Strand, which is used in both pre and post-tensioning is made by winding seven cold drawn wires together on a stranding machine. Six wires are wound in a helix around a centre wire which remains straight. Strands of 19 or 37 wires are formed by adding subsequent layers of wire. Most pre-tensioning systems in New Zealand are based on the use of standard seven wire stress relieved strands conforming to BS3617:"Seven Wire steel strand for Prestressed concrete." With wire tendons and strands, it may be desirable to form a cable to cope with the stressing requirements of large post-tensioning applications. Cables are formed by arranging wires or strands in bundles with the wires or strands parallel to each other. In use the cable is placed in a preformed duct in the concrete member to be stressed and tensioned by a suitable posttensioning method. Tendons whether bars, wires, strands, or made up cables may be used either straight or curved.

i. Straight steel tendons are still by far the most commonly used tendons in pre-tensioned concrete units.
ii. Continuously curved tendons are used primarily in post-tensioning applications. Cast-in ducts are positioned in the concrete unit to a continuous curve chosen to suit the varying bending moment distribution along the members.

6.3 Pre-Tensioning

As discussed, (para 2.2) pre-tensioning requires the tensile force to be maintained in the steel until after the high strength concrete has been cast and hardened around it. The tensile force in the stressing steel is resisted by one of three methods:

a. Abutment method - an anchor block cast in the ground.
b. Strut method - the bed is designed to act as a strut without deformation when tensioning forces are applied.
c. Mould method - tensioning forces are resisted by strong steel moulds.
d.

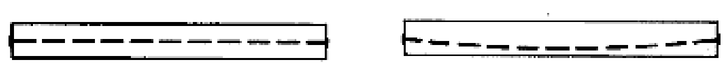


FIG. 6(A) STRAIGHT TENDON FIG. 6(B) CURVED TENDON

FIG. 7(A) ABUTMENT METHOD

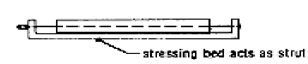


FIG. 7(B) MOULD METHOD – tensioning forces are resisted by strong steel moulds

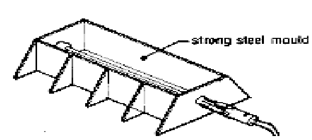


FIG. 7(C) MOULD METHOD

FIG. 7(D) LONG LINE PROCESS

It is usual in pretensioning factories to locate the abutments of the stressing bed a considerable distance apart so that a number of similar units can be stressed at the same time, end to end using the same tendon. This arrangement is called the "Long Line Process". After pouring, the concrete is cured so that the necessary strength and bond between the steel and concrete has developed in 8 to 20 hours. When the strength has been achieved tendons can be released and the units cut to length and removed from the bed.

Post-tensioning systems are based on the direct longitudinal tensioning of a steel tendon from one or both ends of the concrete member. The most usual method of post-tensioning is by cables threaded through ducts in cured concrete. These cables are stressed by hydraulic jacks, designed for the system in use and the ducts thoroughly grouted up with cement grout after stressing has occurred. Cement grouting is almost always employed where post-tensioning through ducts is carried out to:

– Protect the tendon against corrosion by preventing ingress of moisture.
– Eliminate the danger of loss of prestress due to long term failure of end anchorages, especially where moisture or corrosion is present.
– To bond the tendon to the structural concrete thus limiting crack width under overload.

7. Resistance of Prestressed Concrete

All concrete is incombustible. In a fire, failure of concrete members usually occurs due to the progressive loss of strength of the reinforcing steel or tendons at high temperatures. Also the physical properties of some aggregates used in concrete can change when heated to high temperatures. Experience and tests have shown however that ordinary reinforced concrete has greater fire resistance than structural steel or timber. Current fire codes recognise this by their reference to these materials. Prestressed concrete has been shown to have at least the same fire resistance as ordinary reinforced concrete. Greater cover to the prestressing tendons is necessary however, as the reduction in strength of high tensile steel at high temperatures is greater than that of ordinary mild steel.

8. Applications of Prestressing

8.1 General

The construction possibilities of prestressed concrete are as vast as those of ordinary reinforced concrete. Typical applications of prestressing in building and construction are:

1. Structural components for integration with ordinary reinforced concrete construction, e.g. floor slabs, columns, beams.
2. Structural components for bridges.
3. Water tanks and reservoirs where water tightness (i.e. the absence of cracks) is of paramount importance.
4. Construction components e.g. piles, wall panels, frames, window mullions, power poles, fence posts, etc.
5. The construction of relatively slender structural frames.
6. Major bridges and other structures.

Prestressed concrete design and construction is precise. The high stresses imposed by prestressing really do occur. The following points should be carefully considered:

1. To adequately protect against losses of prestress and to use the materials economically requires that the initial stresses at prestressing be at the allowable upper limits of the material. This imposes high

stresses, which the member is unlikely to experience again during its working life.

2. Because the construction system is designed to utilise the optimum stress capability of both the concrete and steel, it is necessary to ensure that these materials meet the design requirements. This requires control and responsibility from everyone involved in prestressed concrete work - from the designer right through to the workmen on the site.

We have seen that considerable design and strength economies are achieved by the eccentric application of the prestressing force. However, if the design eccentricities are varied only slightly, variation from design stresses could be such as to affect the performance of a shallow unit under full working load. The responsibility associated with prestressing work then is that the design and construction should only be undertaken by engineers or manufacturers who are experienced in this field.